UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE M 3- 117

TITLE Automatic Eigenvalue - Eigenvector Program (DOI Only)

TYPE Entire Program

ACCURACY Depends on the condition of the symmetric matrix. Usually 10-11 decimal places.

DURATION
(a) Input program M - 3 about 10 seconds
(b) Input Matrix - about $(n^2 + n)/40$ seconds (depending on number of digits)
(c) Computation time - about $(5\ n^3)\ 10^{-3}$ seconds per iteration, where an iteration means $(n^2 - n)/2$ orthogonal transformations. Most matrices require from 4 iterations (n = 3) to 7 iterations (n = 23) for convergence.

DESCRIPTION This program computes the eigenvalues and eigenvectors of a symmetric matrix of order $n < 24$. It is essentially a combination of library subroutines M-0 (modified), X-1, N-3, R-1 and P-6. The user of this program need only supply the matrix and two parameters. For a description of the mathematical method of computing the eigenvalues and eigenvectors see program M-0.

METHOD OF USE
(1) Read program tape in until it stops.
(2) Insert parameter tape in reader and in the two parameters.
(3) Replace program tape in reader and read in until it stops a second time.
(4) Insert matrix tape in reader and read in matrix.
(5) Replace program tape in reader.

After computation has been completed the remainder of the program will be read in and results will be punched out.

PREPARATION OF PARAMETER TAPE

The user of this program must prepare a parameter tape as follows:

```
00 3K
00 F
00 gF
00 F
00 nF
24 999N
```

The parameter g indicates the number of decimal digits desired in the print out and n is the order of the matrix.

PREPARATION OF MATRIX TAPE

The symmetric matrix (a_{ij}) must be scaled so that $\sum_{i,j} a_{ij}^2 < 1/2$.

Then a_{ij}, for $i \geq j$, is punched as a sign digit followed by up to eleven decimal digits. Element a_{nn} is followed by an N.

EXAMPLE: To diagonalize the 3rd order matrix

$$\begin{bmatrix} 1 & : & : \\ 2 & 4 & : \\ -3 & 5 & 6 \end{bmatrix} = 10^2 \begin{bmatrix} .01 & : & : \\ .02 & .04 & : \\ -.03 & .05 & .06 \end{bmatrix}$$

and print answers to 11 decimal places prepare the tape:

```
00 3K
00 F
00 11F
00 F
00 3F
24 999N
```

Parameter tape

Several spaces

```
+01
+02
+04
-03
+05
+06
N
```

Matrix tape

Notice the order in which the elements are punched. Since the scale factor is 10^{-2} the eigenvalues must be "unscaled" by a factor of 10^{2}. No scaling is necessary for the eigenvectors since they are normalized.

The Printed Results:

For the 3rd order matrix above the results are

+04000000004	4 iterations
-03077303608	
+03841390157	eigenvalues
+10235913450	
N	
+65271954698	
-55145508707	1st eigenvector
+51947519662	
N	
+74655097184	
+58486128172	2nd eigenvector
-31717333989	
N	
-12891407756	
+59483995162	3rd eigenvector
+79343972205	
N	

The first number indicates that 4 iterations (see above) were required for convergence. Ignore the 4 in the eleventh decimal place produced by round-off. The next three numbers are the eigenvalues and following these are the three eigenvectors corresponding to the three eigenvalues, in the same order. The N between eigenvectors permits the output tape to be used as an input tape for another problem if one desires to do so.

ARITHMETIC TEST

In case of an overflow or other type of arithmetic failure the program will stop on a 24 182F order. If started again it will print N(1) and N(14) as 20 consecutive sexadecimal characters and continue. If the first ten characters do not agree with the last ten there has been a failure in Arithmetic.

TAPE MAKE-UP

1. Decimal Order Input No. X-1
   Spaces (parameter tape inserted here)

2. 00 512K
   L5 4F
   L4 4F
   40 5F
   L4 4F
   L0 12L
   40 600F
   L5 4F
   L4 12L
   40 601F
   50 601F
   75 4F
   S5 F
   10 1F Generate parameters
   L4 11L
   40 7F
   50 600F
   75 4F
   S5 F
   10 1F
   L4 11L
   40 13F
   26 999F
   00 F
   00 192F
   00 F
   00 1F
   26 512N

3. 00 31K
   Decimal Number Input No. N-2

4. 00 52K
   Square Root No. R-1

5. 00 62K
Single Column Print No. P-6.

6. 00 20K
80 40F
40 161F
26 161F
00 F
00 1F
00 1F
00 S4
00 S4
00 192F
00 192F Constants
80 S5
00 S5
20 F
00 F
00F 00 0100
0000 0010 J
80 F
00 F
00 F
00 F
J0 S7
74 S7

7. 00 76K
Library Routine M-0

8. 00 800K
L5 10F
L0 1021F
32 1017F
LJ 10F

22 120F
L5 1021F amendment to No. 42
40 10F
L5 1020F
40 2F
26 122F
7L 4095F
LL 4095F
3L 4095F
LL 4095F
L3 6F
32 1023F
26 111F
27 113F

9. 00 850K
L5 800F
40 1015F
L5 L
L4 5L
40 L Plants above amendment beginning at 1015
L0 4L
36 191F
26 L
L5 809F
40 1023F
00 1F
00 1F

10. 00 8K
41 F
L5 28F
40 S7
L5 9F
L4 22F
46 9F
L0 19F
36 14F

```
50 998F
24 31F
00 14K          Generate unit matrix
L5 F
L0 18F
36 8F
L5 F
L4 22F
40 F
11 63F
26 9F
00 S4
00 S4
N0 F
L5 9F
00 191K
50 18F
75 18F
10 1F
L4 7F
00 20F
46 19F
41 191F
26 8F
00 998K
40 192F
50 5F
00 6K
00 F
24 76F
26 85 ON
Spaces          (insert matrix here)
```

11. 80028 400K2
80028 400K3

```
19026 260K1
80028 400K1
L40K2 400K2
80028 4007L
41005 41006
L5016 40007
L50SL 50082
2603F L5082
L4007 46082
L0094 36095
L5007 L4016          Printing
40007 26082          Instructions
L5094 L4005
L4006 4608K
L1000 5008K
2603F 22098
L0093 3608L
L5005 L4017
40005 22096
41005 L5006
L4016 40006
L0017 32092
22096 0F000
L5000 00000
L5000 00000
92302 92081
41007 L5007
L4016 40007
26088 L5007
L0017 3609S
L508K 2608N
92302 92081
41007 2608N
L501F 46094
46094 L500J
00014 46093
26080 00000
2609J 00001
```

RT: 11/4/60

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CODED BY Robert Gregory
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