TITLE	Exponential (D.O.I. or SADOI)
TYPE	Closed, standard entry
NUMBER OF WORDS	21
TEMPORARY STORAGE	0, 1, 2
ACCURACY	Maximum error: 5×10^{-12} (see description)
DURATION	11.3 m seconds (max) (see description)
DESCRIPTION	This routine replaces x, the contents of A before entry by e^x. The quantity x is in the range $-1 < x < 0$. The computation is performed by evaluating a continued fraction expansion for e^x.

The expansion used[1] is:

(1) $\dfrac{e^x}{2} = 1/2 + (x/4) \left(\dfrac{1}{1/2 - x/2 + F/2} \right)$ where

(2) $F = \dfrac{\dfrac{x^2}{4 \cdot 16}}{\dfrac{3}{16} + \dfrac{\dfrac{x^2}{4 \cdot 16 \cdot 16}}{\dfrac{5}{16} + \dfrac{\dfrac{x^2}{4 \cdot 16 \cdot 16}}{\dfrac{7}{16} + \cdots}}}$

[1] N. Macon, "On the Computation of Exponential and Hyperbolic Function Using Continued Fractions", Jour. ACM, pp 262 - 267, October 1955.

This routine is so written that the first 5 terms of
F (eq. 2) are used in computing e^x. It is possible
to modify this, however, by changing the first digit in
instruction 17L, [i.e. the 5 in 58F 00F] to 4, 3, or 2.
The smaller the number used in place of 5, the shorter
the duration of the computation but the greater the
error. A simplified version of the dependency of
speed and accuracy is given by the tables below:

TABLE I

Digit	Max. Error	Duration
5	5×10^{-12}	11.3 m sec.
4	5×10^{-9}	9.7 m sec.
3	5×10^{-6}	8.1 m sec.
2	5×10^{-5}	6.5 m sec.

for $-1 < x < 0$.

It should be noted that the error increases with x
approaching -1. Thus for x in the range
$-1/2 \leq x < 0$ we have the following table:

TABLE II

Digit	Max. Error	Duration
5	5×10^{-12}	Speed same as Table I.
4	5×10^{-11}	
3	5×10^{-10}	
2	5×10^{-8}	

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LOCATION	ORDER	NOTES	PAGE 1
	00 K(S4)		
0	40 F	Set Exit	
	K5 F		
1	42 16L		
	L5 F		
2	10 2F	Form x/2	
	50 F		
3	40 F	Form $x^2/4$	
	7J F		
4	40 1F		
	41 2F		
5	L5 17L		
	40 19L		
6	50 2F		
	85 F		
7	10 3F	Compute F	
	L4 19L		
8	40 20L		
	L5 1F		
9	10 5F		
	66 20L		
10	L5 19L	Test for end of F computation.	
	L0 18L		
11	40 19L		
	L0 18L		
12	32 6L	$1/2 - x/2$	
	L9 F		
13	84 F	$1/2 - x/2 + F/2$	
	40 20L		
14	50 2F		
	L5 F		
15	66 20L		
	8J F	$\dfrac{x/4}{1/2 - x/2 + F/2} + 1/2$	
16	00 1F		
	22 F	e^x	

LOCATION	ORDER		NOTES	PAGE 2
17	58 F		Constant = 11/16	
	00 F			
18	10 F		Constant = 1/16	
	00 F			
19	00 F			
	00 F		Temporary locations	
20	00 F			
	00 F			