TITLE	Matrix Multiplication
TYPE	Complete Program
NUMBER OF WORDS	$198 = 159 + (N12)$
ACCURACY	Up to 12 places. This will depend on the number of places to be punched.
SIZE OF MATRICES	Let A be an $(i \times j)$ matrix and B be an $(j \times k)$ matrix. The number of elements of these matrices must be such that $j k \leq 10,240$ and $j \leq 412$. i is arbitrary.
DURATION	Let ℓ, m, and n be the number of decimal place in the A, B, and product matrices respectively $(4\ jkm + 18\ jk + 80\ k + 500)$ millisec. to input B, and $(3.3\ ijk + 4.ij\ell + 6.6\ ikn + 200\ ik)$ millisec. to input A and perform calculation with output.
TEMPORARY STORAGE	0, 1, 2, 3, 4, 5.
DESCRIPTION	This program will take an $(i \times j)$ matrix A and multiply it by a $(j \times k)$ matrix B yielding an $(i \times k)$ matrix C. No scaling is necessary other than that each element of the matrices be less than one. If overflow occurs, the program records it and punches out the proper element of matrix C accordingly. Each element of C which is larger than one is punched as a signed integral part, a decimal point, and an n place fractional part. Otherwise the element of C is punched as an n place signed fraction. The elements of the matrix C are preceded by three constants i, k, and n, where i indicates the number of rows of C; k, the number of columns, and n, the number of places printed. Each row is terminated by the letter N. Thus if all the elements of C are less than one, the matrix C can be read back into the computer again.

PREPARATION OF MATRIX TAPE

Prepare the matrix A tape by first punching the constants i, j, and n (see note 1) where i denotes the number of rows of A; j, the number of columns of A; and n, the number of decimal digits of c_{ik} punched. These constants are to be separated from each other by a fifth hole character (e.g. space). After these constants, punch the elements of A in a rowwise fashion. Each row of A is terminated with the symbol N (see note 2). Punch the matrix B in a similiar fashion (see note 3).

METHOD OF OPERATION

After the program has been read, a 24 stop occurs. Place the B matrix on the reader. Upon bypassing this step the matrix B is read and placed on the drum (see note 4). A 30 stop follows. Now place matrix A in the reader and bypass the stop. The matrix A is read a row at a time and k elements of matrix C are punched for each row of A (see note 5). When the matrix multiplication has been completed, the program will stop on a 30 order. If one bypasses this stop with the black switch, a new matrix A will be read and multiplied with the old matrix B. If one bypasses this stop by means of the white switch, both a new matrix B and a new matrix A will be read and multiplied together (see note 6).

NOTES

1. The constants i and j need not coincide with the actual number of rows and columns of the matrix but may be less. If this is the case, only the upper left hand (ixj) submatrix is used.

2. If instead of using an N as a row terminating symbol, one used an J, then the program coming upon it will stop. When this stop is bypassed by the black switch, the program will resume input of the matrix. This enables one to have the matrix on several tapes.

3. The last printing constant n to be read governs the number of places printed. The previous printing constants have no effect on the printing of the current matrix.

4. If the FF 03N stop occurs, this signifies that
    SADOI has played back the program improperly.

5. If the FF 03J stop occurs, this signifies that
    the matrix B has been read off the drum incorrectly.
    This stop will occur if the constant j preceding
    the matrix A does not agree with the constant j
    preceding the matrix B.

6. Automatic operation may be achieved by placing
    the black switch in the ignore position. However,
    certain additional characters must be punched between
    matrices, i.e. after the last N of some matrix, yet
    always prior to the three constants on the following
    matrix. These characters have the following meanings;

    N or J: Causes the program to stop on an OF
             instruction. By working the white switch
             up and down, one returns to the point in
             the program which precedes the reading of
             this character.

    F or L: Causes the next matrix input to be treated
             as a B matrix, rather than a new A matrix
             as would be the case if no F or L separates
             the two matrices.

Thus, if one arranges his matrices and inserts the
character F or L properly one may do a sequence of
A·B multiplications with just new A matrices or with
both new A matrices and new B matrices without stopping.
If now an N or J character follows the F or L character,
an OF stop will occur.

3/6/61

DATE ____April 17, 1958____
CODED BY _Ray Polivka_
APPROVED BY _D. E. Muller_

LOCATION	ORDER	NOTES	PAGE 1
	00 6K		
0	42 10L		
	L4 1F		
1	41 2F		
	50 ( )F	by 45, 50, 56, 112	
2	75 134L		
	40 1F		
3	L3 1F		
	32 4L		
4	49 3F		
	L1 2F	Punch indices and results of	
5	32 7L	multiplication	
	L5 1F		
6	00 36F		
	82 4F		
7	10 40F		
	F5 F		
8	40 F		
	L0 135L	by 43, 111, 117	
9	32 10L		
	75 132L		
10	22 2L		
	22 ( )F	by 0, 116	
11	42 16L		
	00 35F		
12	81 4F		
	L0 132L		
13	36 17L	Read in indices preceding the	
	L4 132L	matrices	
14	74 132L		
	00 4F		
15	91 4F		
	36 14L		

LOCATION	ORDER	NOTES
16	S5 F	
	22 ( )F	
17	F0 135L	
	32 19L	if F or L stop on N or J
18	0F F	
	26 12L	
19	00 F	
	F5 19L	
20	26 11L	
	40 145L	Store j
21	F4 131L	
	40 144L	
22	32 22L	
	F5 22L	
23	26 11L	
	40 146L	Store k
24	L4 62L	
	42 133L	
25	00 1F	
	F5 25L	
26	26 11L	
	L5 131L	
27	40 3F	
	41 141L	Clear sum check
28	F5 3F	
	40 3F	Have all j rows been input?
29	L0 144L	
	30 42L	
30	50 192L	
	50 30L	
31	26 153L	
	L0 140L	Stop input if J is encountered.
32	30 32L	
	L5 3F	

LOCATION	ORDER		NOTES
33	40 37L		
	L5 62L		
34	42 35L		
	42 36L		
35	L5 141L		Form sum check
	L4 ( )F	by 34, 40	
36	40 141L		
	L5 ( )F	by 34, 40	
37	86 11F		Read in the rows of matrix B
	00 ( )F	by 33, 39	
38	L5 37L		
	L4 145L		
39	40 37L		
	F5 35L		
40	42 35L		
	42 36L		
41	L0 133L		
	36 35L		
42	26 28L		
	L5 86L		
43	42 8L		
	F5 43L		
44	26 11L		Read index i
	40 152L		
45	L5 44L		
	42 1L		
46	92 131F		
	92 515F		
47	00 1F		
	F5 47L		
48	26 L		Print index i
	92 961F		
49	00 1F		
	L5 23L		

LOCATION	ORDER	NOTES
50	42 1L	
	F5 50L	
51	26 L	Print index k
	F5 51L	
52	26 11L	Read index j
	L4 62L	
53	42 70L	
	S4 118L	
54	42 136L	
	F5 54L	
55	26 11L	Read n
	40 147L	
56	F5 1L	
	40 1L	
57	92 961F	
	F5 57L	
58	26 L	print n
	41 1F	
59	41 150L	
	41 151L	
60	19 39F	
	00 1F	
61	66 132L	
	10 1F	
62	40 F	
	S5 192L	
63	40 150L	
	L5 F	
64	50 151L	
	66 132L	Calculate round off
65	S5 F	
	40 151L	
66	F5 1F	
	40 1F	

LOCATION	ORDER	NOTES		PAGE 5	M 21
67	LO 147L				
	36 69L				
68	51 150L				
	22 60L				
69	7J 140L				
	L4 150L				
70	40 150L				
	S5 F	by 53			
71	40 151L				
	41 4F				
72	50 192L				
	50 72L				
73	26 153L		**Read a row of A**		
	41 5F				
74	L5 137L				
	40 76L				
75	L5 70L				
	42 77L				
76	85 11F				
	00 2560F	by 74, 78			
77	32 77L		**Play back a column of B**		
	40 F	by 75, 79	**(minimum access)**		
78	L4 142L				
	40 142L				
79	F5 76L				
	40 76L				
80	F5 77L				
	42 77L				
81	LO 136L				
	36 76L				
82	L5 30L				
	46 87L				
83	L5 70L				
	42 87L				

LOCATION	ORDER		NOTES
84	L5 138L		
	40 1F		
85	41 148L		This clears Q, should set Q = 1/2 for initial round-off.
	50 148L		
86	41 149L		
	S5 135L		
87	50 F	by 82, 94	
	74 F	by 83, 94	
88	40 F		
	L4 149L		
89	40 149L		
	32 93L		Form $\sum a_{ij} b_{jk}$
90	L4 138L		
	40 149L		
91	L5 F		
	36 127L		
92	L5 148L		
	L0 140L		
93	40 148L		
	L5 87L		
94	L4 139L		
	40 87L		
95	F5 1F		
	40 1F		
96	L0 145L		
	32 86L		
97	S5 F		
	40 143L		
98	92 131F		Preparation for punching
	92 513F		
99	L5 148L		
	36 128L		
100	92 706F		Punch minus sign
	L1 148L		

LOCATION	ORDER	NOTES
101	40 148L	
	L1 149L	
102	40 149L	
	L1 143L	
103	40 143L	
	L5 151L	
104	50 143L	
	74 140L	Add in the least significant and
105	L4 150L	most significant parts of the round-off.
	50 149L	
106	74 140L	
	40 F	
107	S5 F	
	40 149L	
108	L5 F	
	L4 148L	
109	40 148L	
	L3 148L	
110	36 115L	
	41 2F	
111	L5 86L	
	42 8L	
112	L5 85L	
	42 1L	
113	00 1F	Print the overflow part of
	F5 113L	$\sum a_{ij}\, b_{jk}$
114	26 L	
	92 643F	decimal point
115	49 2F	
	L5 53L	
116	42 10L	
	L5 55L	Print the fractional part of
117	42 8L	$\sum a_{ij}\, b_{jk}$
	50 149L	

LOCATION	ORDER		NOTES
118	27 8L		
	F5 5F		
119	40 5F		
	LO 146L		Have all k columns been played back?
120	36 121L		
	26 75L		
121	L5 141L		
	LO 142L		
122	40 F		Do the sum checks agree?
	L3 F		
123	36 129F		
	FF 61F		
124	F5 4F		
	40 4F		Have all i rows of A been read?
125	LO 152L		
	30 42L		→Black switch – Read a new matrix A
126	26 72L		
	22 19L		← White switch – Read new matrices B and A
127	F5 148L		
	26 93L		
128	92 642F		print +
	22 103L		
129	92 770F		print N
	92 131F		
130	41 142L		
	26 124L		
131	86 11F		Constants
	00 2559F		
132	00 F		
	00 10F		
133	75 141L		
	L4 ( )F	by 24	
134	01 1146F		
	F1 1147F		10^{-2}

LOCATION	ORDER		NOTES
135	00 F		
	00 3F		
136	S2 77L		
	40 F		
137	85 11F		
	00 2560F		
138	80 F		
	00 F		
139	00 1F		
	00 1F		
140	00 F		
	00 1F		
141	00 F		Record sum check
	00 F		
142	00 F	by	Playback sum check
	00 F		
143	00 F		Least significant part of $\sum a\, b$
	00 F		
144	00 F		
	00 F	by 21	
145	00 F		
	00 F	by 20	j
146	00 F		
	00 F	by 23	k
147	00 F		
	00 F		n
148	00 F		Overflow of $\sum ab$
	00 F		
149	00 F		Most significant part of $\sum ab$
	00 F		
150	L3 F		Most sign. part
	30 19L		of round-off
151	FF 60F		Least sign. part
	22 19L		of round-off

Most sign. part of round-off / Least sign. part of round-off — Originally "Did SADOI playback correctly?"

LOCATION	ORDER	NOTES	PAGE 10	M 21
152	00 F			
	00 F	i		
	(N12)			
	24 156N			