

UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE A 5 - 138

TITLE Complex Number Arithmetic (DOI or SADOI)

TYPE Interpretive routine with 18 orders, entered as a closed routine, left by an 8J or 8N order.

NUMBER OF WORDS 248

TEMPORARY STORAGE 0, 1, 2, 3

ACCURACY About 9 decimals

PARAMETERS During input location 3 must contain $n \times 2^{-39}$ where n , $n+1$, and $n+2$ is the location of the floating decimal accumulator.

METHOD OF USE The reader is referred to the summary description of the floating decimal arithmetic routine for all details of use. The differences are given here.

82 n } Transfer control if the real part of F is ≥ 0 .

83 n }

88 0 Replace F by one number read from the tape punched as the real part, imaginary part, and exponent. Following the exponent a space or carriage return should be punched. For example:
 $(.8971 - i .0632) \times 10^{10}$ would be punched as
+ 8971 - 0632 + 10 space.
Printing follows the same form.

8N n Transfer control to the ordinary order on the right hand side of n.

b2 n Replace e_p, c_p by $e_p + 2, c_p + 1$ then transfer control, etc.

b3 n

This last modification was necessitated by the fact that two memory registers are used for handling a single number. For this reason one will in general wish to step addresses referring to numbers by two.

USE OF AUXILIARY ROUTINES

Auxiliary routines are entered in the same way as in the floating decimal routine. If control is transferred to word 19 or 29 the same effect is produced as in the case of the floating decimal routine. Several auxiliaries are available for use with the complex number routine. These are included within the main body of the routine and may be entered by means of 8J orders with the following given addresses relative to the routine:

<u>Address of 8J Order</u>	<u>Auxiliary</u>
42	Replace F by the complex conjugate of F.
210	Replace F by the modulus of F.
225	Replace F by the principal value of the square root of F.

If the last two auxiliaries are not needed one may omit word 210 and all words following it from the routine since they are used only by the auxiliaries.

HANDLING OF NUMBERS

Each number is represented in the form $(A + 1B) \times 10^p$ where $1 > |A| + |B| \geq 1/10$ approximately, and $512 > p \geq -512$. In the floating decimal accumulator A/2 is in 83 as a fraction, p + 512 is in 183 as an integer and B/2 is in 283 as a fraction. When a number is packed for storage in another part of the memory it uses only two registers and the exponent p is divided equally between the two.

The number zero is an exception and is represented as a number with A = B = 0 and p = -512. Thus zero is represented in the same way as the machine representation of zero. When capacity is exceeded a stop is encountered at word 102 of the routine. This stop is a 34 102L order. The b-registers are located from word 191 to word 198 of the routine.

RT: 10/13/60

DATE	June 10, 1955
CODED BY	<i>DE. Muller</i>
APPROVED BY	<i>J. Nash</i>

LOCATION	ORDER	NOTES	PAGE 1	A 5
0	00K(A5) 00 59F L0 199L	Special orders for entry		
1	L0 200L 40 2L	Plant either 50 n S5 20F or L5 nF 00 20F in 2L		
2	50 500F S5 20F	Select order to be obeyed		
3	40 3F 32 183L	Adjust address if necessary		
4	46 8L L4 209L	Insert address in 8L and 1 + address in 7L		
5	46 7L 10 12F			
6	L4 43L 46 11L	Form switch order		
7	50 F 41 1F			
8	L5 F 40 F	Unpack $(A + iB) \times 10^p$ placing A in 0, B in 2, and $p + 512$ in 1		
9	00 5F 42 1F			
10	S5 F 40 2F			
11	26 F 75 201L			
12	00 4F 40 S3	$A \times \frac{10}{16} \times 2^4$ in S3		
13	50 2S3 75 201L			
14	00 4F 40 2S3	$B \times \frac{10}{16} \times 2^4$ in 1S3		
15	22 23L 7J F			
16	L4 S3 40 S3	$A \times 10^{p-q} + A'$ in S3		

LOCATION	ORDER	NOTES	PAGE 2
17	50 2F 7J F	- $B \times 10^{p-q} + B'$ in 2S3	
18	L4 2S3 40 2S3		
19	50 S3 L3 S3	- Test for $ A + B = 0$	
20	L2 2S3 36 97L		
21	L0 148L 36 25L	or $ A + B > 1/2$	
22	L4 150L 32 11L	- or $ A + B < 1/20$	
23	26 29L K1 F		
24	L4 1S3 26 28L	- $p - 1$ in 1S3	
25	7J 32L 40 S3		
26	50 2S3 7J 32L	- $A \times .1$ in S3 $B \times .1$ in 2S3	
27	40 2S3 F5 1S3	$p + 1$ in 1S3	
28	40 1S3 26 19L	Recycle	
29	L5 2L 36 1L	- Re-enter routine	
30	L4 202L 22 1L		
31	7L 4095F LL 4095F		
32	00 F 00 1000 0000 0000 J		
33	00 F 00 1000 0000 003J		

LOCATION	ORDER	NOTES	PAGE 3
34	00 F 00 1000 0000 00J		
35	00 F 00 1000 0000 0J		
36	00 F 00 1000 0000 J		
37	00 F 00 1000 000J	Constants	
38	00 F 00 1000 00J		
39	00 F 00 1000 0J		
40	00 F 00 1000 J		
41	00 F 00 512F		
42	L1 283 22 18L		
43	00 172L 00 31L		
44	L1 F 26 60L	b 0	
45	L1 F 22 93L	b 1	
46	L5 204L 22 170L	b 2	
47	L5 204L 26 171L	b 3	
48	J0 104F 22 61L	b 4	
49	L5 F 26 95L	b 5	
50	L1 1F 26 83L	b 6	

LOCATION	ORDER	NOTES	PAGE 4 A 5
51	L1 41L 22 72L	b 7	
52	81 4F 26 110L	b 8	
53	L5 132L 22 135L	b 9	
54	41 1F 22 164L	b K	
55	L5 8L 26 98L	b S	
56	L5 3F 22 182L	b N	
57	L5 3F 22 6L	b J	
58	92 131F 26 133L	b F	
59	L5 8L 22 181L	b L	
60	40 F L1 2F	Form - A and - B	
61	40 2F L5 1S3		
62	L0 1F 50 F	Compare exponents of the two numbers to be added	
63	32 69L 40 3F		
64	50 2S3 L5 2F		
65	40 2S3 S5 F		
66	40 2F 50 S3	Interchange N(2) and N(2S3). Put N(1) in LS3 and N(S3) in Q if N(LS3) < N(1). Put	
67	L5 F 40 S3	N(LS3) - N(1) in A	

LOCATION	ORDER	NOTES	PAGE 5
68	15 1F 40 183		
69	11 3F 14 43L		
70	42 15L 42 17L	Select power of 10	
71	10 206L 36 29L	Test to see ratio of two numbers is $> 10^9$	
72	22 15L 14 1F		
73	14 183 40 183	Put p - q in 183	
74	50 F 7J 283	A x B' in 3	
75	40 3F 50 2F		
76	73 83 14 3F	$(A' \times B + A \times B') \times 2$ in 3	
77	00 1F 40 3F		
78	50 F 7J 83	A x A' in 83	
79	40 83 50 2F		
80	71 283 14 83	$(A \times A' - B B') \times 2$ in 83	
81	00 1F 40 83		
82	15 3F 22 18L	Standardize result	
83	F4 207L 40 1F	$1825 - (p + 512)$ in 1	
84	50 F 7J F	A^2 in 3	
85	40 3F 50 2F		

LOCATION	ORDER	NOTES	PAGE 6
86	75 2F	$\frac{(A^2 + B^2) \times 4}{(A^2 + B^2) \times 4} \text{ in } 3$	
	L4 3F		
87	00 2F		
	40 3F		
88	50 F		
	75 32L		
89	66 3F	$\frac{Ax - 1}{(A^2 + B^2) \times 4} \text{ in } 0$	
	85 F		
90	40 F		
	50 2F		
91	71 32L	$\frac{-Bx - 1}{(A^2 + B^2) \times 4} \text{ in } 2$	
	66 3F		
92	85 F		
	40 2F		
93	26 51L	Go to multiply	
	40 S3		
94	L1 2F	Form - A and - B	
	26 96L		
95	40 S3	Form A and B	
	L5 2F		
96	40 2S3	And place in S3, 2S3, and p in 1S3	
	L5 1F		
97	40 1S3		
	26 29L		
98	46 108L	Plant n and n + 1	
	L5 7L		
99	46 109L		
	L5 1S3		
100	32 101L	Test for $p < -512$	
	J1 203L		
101	26 108L	Test for $p \geq + 512$	
	L0 207L		
102	34 102L		
	50 208L		

LOCATION	ORDER	NOTES	PAGE 7
103	J0 S3 S5 F	Eliminate last five bits of A and place in 0	
104	40 F L5 2S3		
105	F4 203L 40 2F	Round off for B and place in 2	
106	L5 1S3 50 2F		
107	10 5F L4 F		
108	40 F S5 F	Pack $(A + i B) \times 10^D$ into two registers	
109	40 F 26 29L		
110	40 2F 49 1F		
111	L5 2F 00 39F	Store sign and set $N(1)$ to $\pm 1/2$	
112	32 113L L1 1F		
113	40 1F 23 117L		
114	50 1F 7J 32L	$\pm 1/2 F \times 10^{-n}$ in 1	
115	40 1F 50 1F		
116	75 2F 00 39F	$1/2 F \times 10^{-n}$ in 0	
117	L4 F 40 F		
118	81 4F 40 2F	Read digit and test for sign	
119	L0 203L 32 120L		

LOCATION	ORDER	NOTES	PAGE 8
120	26 114L		
	L1 124L		
121	40 124L	Binary switch for real or imaginary part	
	32 123L		
122	L5 F		
	40 S3	Real part	
123	22 110L		
	L5 F		
124	40 2S3	Imaginary part	
	41 F		
125	74 203L		
	00 4F	Exponent formation and test for fifth hole	
126	91 4F		
	36 125L		
127	22 127L	Waste	
	L1 2F		
128	L4 203L	Test sign of exponent	
	36 130L		
129	S1 F		
	22 130L	Adjust sign	
130	S5 F		
	L4 41L		
131	40 1S3	Store in 1S3	
	26 19L	Go to standardize	
132	00 F		
	00 2F		
133	50 3F		
	00 19F	Set n columns	
134	42 132L		
	47 132L		
135	26 29L		
	F0 209L	Test for new row	
136	36 138L		
	92 131F		

LOCATION	ORDER	NOTES	PAGE 9
137	92 513F 00 24F	Form new row	
138	46 132L 50 S3	Sign of A in a_0	
139	S7 F L0 208L	Round off	
140	10 39F 00 40F		
141	74 31L L4 203L	Form 10 + sign to print K or S in following loop	
142	26 143L 75 203L		
143	00 36F 82 4F		
144	10 40F L5 8L	Print loop	
145	L0 122L 46 8L		
146	32 142L 26 147L	Waste	
147	92 961F L1 48L		
148	40 48L 32 149L	Binary switch for real or imaginary part	
149	22 151L L5 3F		
150	46 8L 50 283	Reset counter and print imaginary part	
151	26 139L L5 183		
152	L0 41L 40 F		
153	32 154L 92 708F	Test for sign of exponent	

LOCATION	ORDER	NOTES	PAGE 10
154	26 155L 92 644F		
155	L7 F 10 38F		
156	66 203L 40 F	-Form digits of exponent	
157	67 203L 40 1F		
158	10 1F S3 F	Test for two digit exponent	
159	36 161L S5 F		
160	00 36F 82 4F	-Print first of 3 digits	
161	L5 1F 00 4F		
162	L4 F 00 31F	-Print last two digits	
163	82 8F 92 965F		
164	26 29L L5 3F	Test for $b \neq 8$	
165	36 178L 46 1F		
166	50 1F 75 33L	-Make $A = \frac{n}{2} \times 10^{-2}$	
167	00 18F 40 S3		
168	41 2S3 F5 2S3		
169	F4 41L 40 1S3	-Make $B = 0$ and $p = 513$	
170	26 19L L0 200L	Set instruction counter	

LOCATION	ORDER	NOTES	PAGE 11
171	40 1F 15 3F		
172	46 1F 32 175L	- Test for $b \neq 0$	
173	15 83 32 174L		
174	26 29L 15 1F	- Transfer control if $A \geq 0$	
175	22 1L 15 191L		
176	14 205L 40 191L	- Advance b register	
177	00 28F 22 173L	Test for transfer	
178	10 20F 42 1F	n in 1	
179	11 1F 50 191L		
180	14 190L 40 (191)L	- Set B register	
181	26 29L 46 (191)L		
182	26 29L 46 183L		
183	22 F 50 3F		
184	01 3F 14 179L		
185	42 175L 42 176L		
186	42 180L 42 181L	- Set B addresses	
187	42 188L J0 199L		

LOCATION	ORDER	NOTES	PAGE 12
188	11 3F 15 191L		
189	84 F 26 4L		
190	80 F 00 2048F		
191	80 F 00 F		
192	80 F 00 F		
193	80 F 00 F		
194	80 F 00 F	-b register	
195	80 F 00 F		
196	80 F 00 F		
197	80 F 00 F		
198	80 F 00 F		
199	LL 4094F 4K 4076F		
200	5S F S5 F		
201	50 F 00 F		
202	5S 1F S5 F		
203	00 F 00 10F		
204	50 F S5 20F	- Constants	

LOCATION	ORDER	NOTES	PAGE 13 A 5
205	00 2F		
	00 1F		
206	00 172L		
	00 42L		
207	00 F		
	00 1024F		
208	LL 4095F		
	LL 4064F		
209	00 1F		
	00 F		
210	22 210L		
	F5 210L		
211	26 213L	- To form modulus enter closed modulus subroutine at 213	
	L5 2F		
212	40 S3		
	23 18L		
213	42 223L		
	50 S3		
214	7J S3		
	40 1F		
215	50 2S3	- $A^2 + B^2$ in 1	
	7J 2S3		
216	L4 1F		
	40 1F		
217	41 F		
	22 218L	Waste	
218	42 223L		
	51 1F		
219	10 1F		
	SJ F		
220	40 2F		
	50 F		
221	L5 1F	- Square root subroutine (see R 1 - 116)	
	66 2F		

LOCATION	ORDER	NOTES	PAGE 14	A 5
222	S5 F			
	L0 2F			
223	10 1F			
	32 F			
224	L4 2F			
	26 220L			
225	22 225L	To form complex square root form modulus with 213		
	F5 225L			
226	26 213L			
	L5 2F			
227	L6 S3	$\sqrt{A^2 + B^2} + A $ in 1		
	40 1F			
228	41 F	Form $\sqrt{A^2 + B^2} + A $ in 2		
	F5 228L			
229	26 218L			
	L7 2S3			
230	50 F	$\frac{ B }{N(2)}$ in 1		
	66 2F			
231	S5 F			
	40 1F			
232	F5 1S3			
	L4 41L			
233	10 1F	Adjust exponent and test for p odd		
	40 1S3			
234	SJ F			
	32 243L			
235	L5 S3			
	32 238L			
236	L5 1F	If A is negative put N(1) in 2 and N(2) in 1		
	50 2F			
237	40 2F			
	S5 F			
238	40 1F			
	F5 2F			

LOCATION	ORDER	NOTES	PAGE 15	A 5
239	10 1F 40 S3			
240	F5 1F 10 1F	- Put result in floating accumulator		
241	50 2S3 40 2S3			
242	S5 F 36 19L	- Make sign of imaginary part same as sign of B		
243	26 42L 50 1F			
244	7J 247L 40 1F			
245	50 2F 7J 247L	- Multiply by $\sqrt{-1}$		
246	40 2F 26 235L			
247	00 F 00 3162 2776 6017 J	$\sqrt{-1}$		