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COUNTEREXAMPLE TO A CONJECTURE OF FUJII, KASAMI AND NINOMIYA

by

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ABSTRACT

In a recent paper [1], Fujii, Kasami and Ninomiya presented a procedure for the optimal scheduling of a system of unit length tasks represented as a directed acyclic graph on two identical processors. The authors conjecture that the algorithm can be extended to the case where more than two processors are employed. This note presents a counterexample to that conjecture.

[1] Fujii, M., .T. Kasami and K. Ninomiya, "Optimal Sequencing of Two Equivalent Processors," SIAM J. Appl. Math., Vol. 17, No.4, July 1969, pp. 784-789.

Consider a system consisting of a set of tasks $T = \{T_i\}$ $1 \leq i \leq n$, and a directed graph G_p representing the precedence relations * among the n tasks. Each task is assumed to require exactly one unit of time. Fujii, Kasami and Ninomiya [1] have presented the following scheduling algorithm, which is optimal for the case of two processors. The algorithm is restated for the case of an arbitrary number of processors:

1. Partition T into a minimal number of subsets, subject to the following restrictions:

   a) The cardinality of each subset must not exceed p, the number of available processors.

   b) All of the members of any subset β in the partition must be compatible (i.e. if $T_i, T_j \in \beta$, $T_i \not> T_j$ and $T_i \not< T_j$).

   Let P_1 be the partition be so formed.

2. Form a sequence $\beta_1, \ldots, \beta_k$ of subsets of T, which will correspond to the execution sequence of an optimal schedule, and a sequence of partitions $P_1, P_2 = P_1 - \beta_1, P_3 = P_2 - \beta_2, \bullet$ **,

   $$P_k = P_{k-1} - \beta_{k-1}, P_{k+1} = \emptyset$$

   as follows:

   a) Select and remove from P_i a subset β_i of T in which every element of β_i is maximal (has no predecessors in any remaining subset of P_i). Terminate if $P_i = \emptyset$, the empty partition.

   b) If no such subset exists, form a new partition, P_i', in which such a subset does exist. This is always possible for p=2 by Lemma 1 of the paper [1]. By the Lemma, $|P_i'| = |P_i|$. Go to step 2a.

   c) Form $P_{i+1} = P_i - \beta_i$. Go to step 2a.

* We will use the notation $T_i < T_j$ (or $T_j > T_i$) to indicate the relation "T_i preceeds T_j".

In this algorithm, the cardinality of P decreases by 1 at each iteration, so that the sequence $\beta_1, \ldots, \beta_k$ has $k=|P_1|$, which is also a lower bound for the total execution time. Hence this is an optimal sequence.

The following counterexample shows that step 2.b is not always possible when there are 3 processors:

A minimal partition', P, is $\{\{T_1,T_5,T_6\}, \{T_4,T_2,T_3\}\}$, $|P| = 2$. However, the best time which can be achieved is 3, corresponding to a partition (e.g.1 $P = \{\{T_1,T_4\}, \{T_2,T_3,T_5\}, \{T_6\}\}$ with $|P| = 3$.

Hence, Lemma 1 does not generalize for $p > 2$ and the presented algorithm is not extendable to 3 processors.