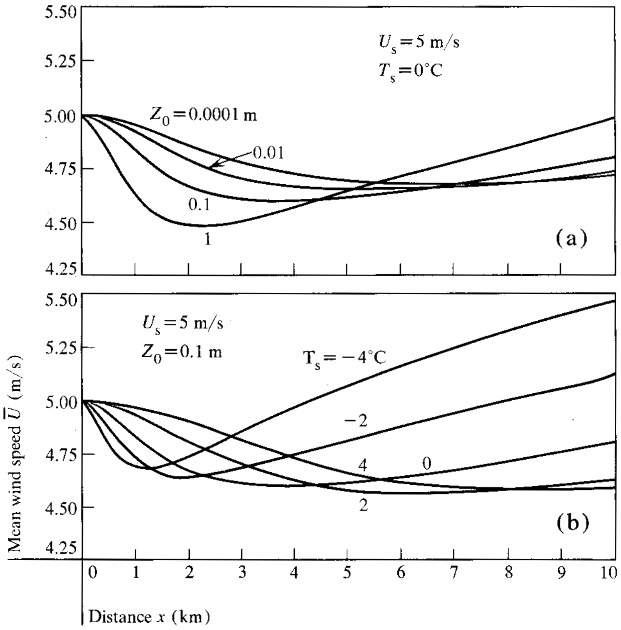

Figure 8 Horizontal variation of effective mean wind with (a) ground roughness and (b) atmospheric stability.

creases downwind. The initial decrease is due to ground reflection which causes large concentrations in the lower layer. When the vertical concentration distribution becomes uniform far downwind, the mean wind increases because a large portion of the concentration is in the upper layer where the wind speed is higher. For increasing ground roughness, the effective mean wind changes more rapidly, the location of its minimum is closer to the stack, and the plume spreads more widely.

The variation of effective mean wind is important because the location of the maximum concentration may not be coincident with that of the maximum relative concentration, which is determined by the effective mean wind. The relation can be determined from

$$\frac{\partial}{\partial x}\left(\frac{\overline{C}\,\overline{U}}{Q_c}\right) = \frac{1}{Q_c}\left(\overline{U}\frac{\partial \overline{C}}{\partial x} + \overline{C}\frac{\partial \overline{U}}{\partial x}\right). \tag{36}$$

In general, there is only one relative maximum for $\overline{C}$ and $\overline{C}\,\overline{U}/Q_c$ and one minimum for $\overline{U}$. The maxima coincide only where $\partial \overline{U}/\partial x = 0$. If $\overline{U}$ is monotonically increasing, as is usually assumed, $\overline{C}_{\max}$ always occurs closer to the stack than $(\overline{C}\,\overline{U}/Q_c)_{\max}$.

Figure 8(b) shows the effects of atmospheric stability on the mean wind distribution. The unstable atmosphere ($T_s \equiv \theta_s - \theta_0 = -4$°C) has an effect similar to that of pronounced ground roughness.

Discussion

Our numerical model can accommodate temporal and spatial variation of meterological factors, feedback of the plume, inhomogeneous terrain[11], and other features[12]. Extension of the model to include multiple source emission, urban factors, and complicated topography is possible and would be advantageous.

From this study we have learned that numerical methods can offer quantitative results for predicting atmospheric dispersion of stack effluents, although questions may be raised about the validity of the empirical formulas for the eddy coefficients. The agreement between theory and experiments for several atmospheric conditions and ground roughness values shows that the gradient-transfer theory has reasonable validity for practical applications. Certainly more study on this theory is needed. A systematic study of the relation between the stability parameters used in theory and the stability categories used in experiments is also urgently needed.

Our results indicate that ground roughness is an important parameter that may resolve seemingly inconsistent data. We also found that the effect of wind speed is small, at least in the neutrally stable atmosphere case. The effect of source height is not completely clear. The source height used in our computation is 100 m, which is the same as that of the Brookhaven data[2]. On the other hand, Pasquill-Gifford's data[3], which are here used for tall stacks, are actually based on a surface source. For a surface source, the effective mean wind increases monotonically downwind. If one estimates the vertical standard deviation from the ground-level concentration with constant wind speed, one obtains a larger deviation at distances far downwind than for an elevated source. The effect of the source height may be to change the slope of the standard deviation curve, and ground roughness may cause it to be displaced. Ground roughness is definitely believed to be an important factor in the discrepancy.

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Appendix—Three-dimensional interpretation of the two-dimensional advection-diffusion equation

by W. E. Langlois

The realism of Shir's results may at first appear paradoxical. His numerical model is two-dimensional, so that the simulated stacks are actually line sources, whereas real-atmosphere stacks are (essentially) point sources. The physical dimensions of the source strengths aren't even the same for the two cases. Moreover, even in a unidirectional wind field, atmospheric turbulence acts to diffuse the plume in the three directions, not two. This point is especially important in view of the basic differences between two- and three-dimensional turbulence, which arise from the absence of vortex stretching in the two-dimensional case.

Nevertheless, we can show that quantitative information about a stack plume can, under some circumstances, be deduced from Shir's two-dimensional calculation. If the ambient wind is fairly strong, turbulent crosswind transport is less pronounced than either the downwind advective transport or the vertical transport induced by plume buoyancy. This intuitive argument can be codified because Shir's use of eddy diffusion theory, rather than the statistical theory of turbulence, diminishes the importance of the fundamental difference between turbulence in two and that in three dimensions.

For a divergence-free wind field and negligible molecular diffusivity, the instantaneous and local concentration C obeys the diffusion equation

$$\frac{\partial C}{\partial t} + \nabla\cdot C\mathbf{V} = Q_c(x,y,z)\,, \qquad \text{(A1)}$$

where $\mathbf{V}$ is the instantaneous local velocity. Assume that for $0 < x < L$ the plume stays within a crosswind strip of width $2Y$ centered on the stack, as illustrated in Fig. A1. Let

$$C(x,y,z) = \overline{C}(x,z) + c(x,y,z)\,, \qquad \text{(A2)}$$

where

$$\overline{C} = \frac{1}{2Y}\int_{-Y}^{Y} C\,dy. \qquad \text{(A3)}$$

Using analogous decompositions for $\mathbf{V} = (U,V,W)$ and Q_c, we obtain

$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial}{\partial x}(\overline{U}\,\overline{C}) + \frac{\partial}{\partial z}(\overline{W}\,\overline{C}) = -\frac{\partial}{\partial x}(\overline{uc}) - \frac{\partial}{\partial z}(\overline{wc}) - \frac{1}{2Y}[VC]_{-Y}^{Y} + \overline{Q}_c. \qquad \text{(A4)}$$

Since the plume is confined to the strip $|y| < Y$, the concentration is zero at $y = \pm Y$. Also, for the stiuation treated by Shir, $\overline{W}$ and $\partial\overline{U}/\partial x$ both vanish. Hence

$$\frac{\partial \overline{C}}{\partial t} + \overline{U}\frac{\partial \overline{C}}{\partial x} = -\frac{\partial}{\partial x}(\overline{uc}) - \frac{\partial}{\partial z}(\overline{wc}) + \overline{Q}_c. \qquad \text{(A5)}$$

We can now replace the customary definitions of horizontal and vertical eddy diffusivity K_h and K_c by

$$\overline{uc} = -K_h\frac{\partial \overline{C}}{\partial x} \quad \text{and} \quad \overline{wc} = -K_c\frac{\partial \overline{C}}{\partial z}. \qquad \text{(A6)}$$

This replacement yields

$$\frac{\partial \overline{C}}{\partial t} + \overline{U}\frac{\partial \overline{C}}{\partial x} = \frac{\partial}{\partial z}\left(K_c\frac{\partial \overline{C}}{\partial z}\right) + \frac{\partial}{\partial x}\left(K_h\frac{\partial \overline{C}}{\partial x}\right) + \overline{Q}_c\,, \qquad \text{(A7)}$$

which is Shir's Eq. (4), generalized to the case of horizontally variable eddy diffusivity.

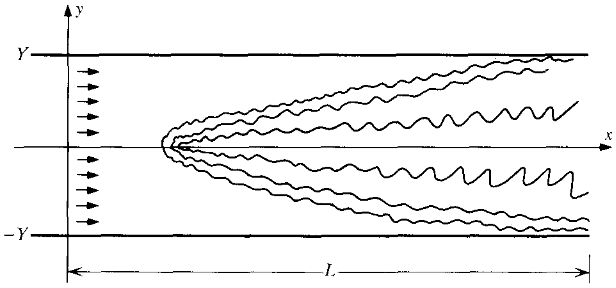

Figure A1 Horizontal plume cross section.

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