Comment on "A Topological Theory of Domain Velocity in Semiconductors"

Abstract: Recently Gunn presented a simple formula for the domain velocity in a "diffusion-controlled" semiconductor, based on topological arguments. It is shown that these arguments are generally not valid. The apparent agreement between Gunn's formula and Hauge's computer simulation is briefly discussed.

Introduction

Recently Gunn[1], using topological arguments, concluded that the velocity for a "diffusion-controlled" domain is

$$v_D = v_0 - \frac{en_0}{\epsilon}\frac{dD(E_2)}{dE}, \tag{1}$$

where v_0 is the outside drift velocity, n_0 is the net donor density, $D(E)$ is the field-dependent diffusion coefficient, and E_2 is the electric field for which $v(E_2) = v_0$ in the negative slope region of the velocity-field characteristic [see Fig. 1(a)]. Butcher et al.[2] have arrived at another analytical expression for v_D, written in terms of integrals around a closed trajectory in the phase plane (E, n) (where n is the carrier density). Preliminary attempts to show the equivalence between the two expressions have failed.[1,3] Hence, Hauge[3] performed a computer simulation of domain formation and propagation for different piecewise linear shapes of $D(E)$, and concluded that the calculated domain velocities were consistent with (1).

Jones et al.[4] have proved analytically that Gunn's condition, from which the domain velocity is derived, is invalid. They perform an exhaustive computer study of the McCumber and Chynoweth model to reach all the pertinent topological classes of solutions. We intend to show that these solutions can be classified by topological arguments alone. Our analysis shows that Gunn's topological arguments are false.

Analysis

Steadily propagating solutions in the "diffusion-controlled" case are described by Poisson's equation

$$\frac{dE}{dx} = \frac{e}{\epsilon}(n - n_0), \tag{2}$$

and the equation (see Ref. 5)

$$\frac{dn}{dx} = \frac{1}{D}\left[(v - v_0)n - \left(c + \frac{e}{\epsilon}\frac{dD}{dE}n\right)(n - n_0)\right], \tag{3}$$

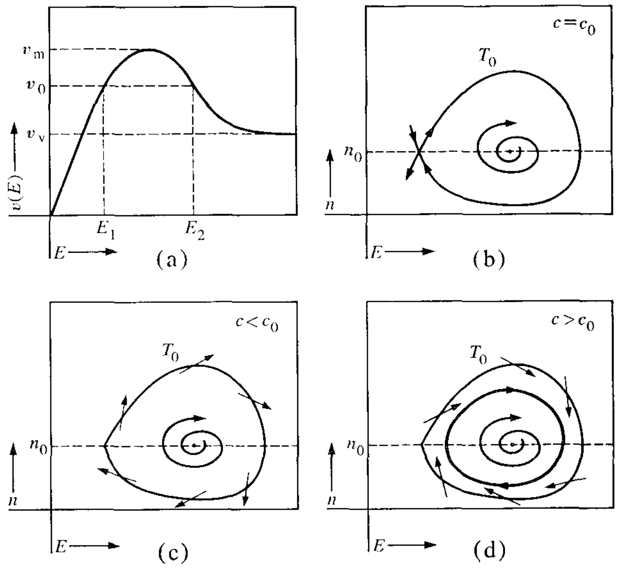

Figure 1 (a) Average velocity v of electrons as a function of electric field; (b) trajectories in phase plane when $c = c_0$, i.e. a high-field domain exists; (c) trajectories for $c < c_0$; (d) trajectories for $c > c_0$, showing the limit cycle (heavy line).

where c is the "excess velocity", i.e. $v_D = v_0 + c$. Equations (2) and (3) are equivalent to equations (7) in Gunn's paper. Division of (3) by (2) yields

$$\frac{dn}{dE} = \frac{\epsilon}{e}\frac{1}{D}\left[(v - v_0)\frac{n}{n - n_0} - c - \frac{e}{\epsilon}\frac{dD}{dE}n\right], \tag{4}$$

which is the convenient form for phase plane investigation.

The singular points for the system, with a saturated velocity-field characteristic as shown in Fig. 1(a), are $S_1 = (E_1, n_0)$ and $S_2 = (E_2, n_0)$ [see Fig. 1(a) for notation]. The nature of the singular points is established by linearizing (2) and (3), and solving the characteristic equation. The roots are ($i = 1, 2$):

$$\left[\lambda_{i\pm} = -\frac{1}{2D_i}\left(c + \frac{e}{\epsilon}n_0\frac{dD_i}{dE}\right)\right. \\ \left.\pm\left[\frac{1}{4D_i^2}\left(c + \frac{e}{\epsilon}n_0\frac{dD_i}{dE}\right)^2 + \frac{e}{\epsilon}\frac{n_0}{D_i}\frac{dv_i}{dE}\right]^{\frac{1}{2}}\right. \tag{5}$$

It can be shown that S_1 is a saddle point, and S_2 a node or a focus depending upon the value of c. Defining

$$c_D = -\frac{e}{\epsilon}n_0\frac{dD_2}{dE}, \tag{6}$$

and referring positive direction in the phase plane to increasing values of x, we find that S_2 is *stable* if $c > c_D$, and *unstable* if $c < c_D$.

As mentioned by Gunn, a high-field domain is obtained for the value $c = c_0$ which yields the separatrix emanating from S_1, encircling S_2, and ending on S_1 [denoted T_0 in Fig. 1(b)]. Gunn also points out that all trajectories crossing T_0 will flow out of T_0 for $c < c_0$, and into T_0 for $c > c_0$ [as shown in Fig. 1(c) and (d)]. Then, according to Gunn, this change from inward to outward flow of trajectories as c passes through c_0 shows that, for $c = c_0$, the trajectories immediately inside T_0 must be closed curves. And further, citing Gunn: "Because all the functions entering the problem are continuous, no limit cycle can exist inside T_0 under these conditions. The nest of closed curves must therefore continue inward to enclose a singular point, which is thus seen to be a *center* when $c = c_0$."

The fallacy of Gunn's reasoning is his statement that a limit cycle cannot exist. His conclusion is that c_0 must equal c_D. Let us assume that this is not true, i.e. $c_0 \neq c_D$. Then the nature of S_2 will not change when c is varied through c_0. First, let us assume that $c_0 < c_D$, which means that S_2 is *unstable* for $c = c_0$, as shown in Fig. 1(b). When $c < c_0$, the trajectories emanating from the source S_2 may flow continuously outward and cross T_0. This is the situation shown in Fig. 1(c). When $c_0 < c < c_D$, trajectories still flow out of S_2, while trajectories crossing T_0 flow inward. Then, according to the Poincaré-Bendixon theorem, (at least) one stable limit cycle must exist inside T_0, as shown in Fig. 1(d). Note that in Figs. 1(b) through (d), S_2 is shown as a focus. However, the topological arguments are also valid if S_2 is a node.

The limit cycles shrink when c increases. This is shown by considering the limit cycle T corresponding to a given value of c. For a still larger c, say c', the trajectories crossing the limit cycle T flow inwards. Hence, a limit cycle T' must exist *inside* T. When c passes through c_D, the limit cycle disappears, because S_2 then becomes a stable focus.

When c approaches c_0, the limit cycle approaches T_0, but T_0 itself is not a limit cycle according to the usual definition, since it does not correspond to a periodic solution.

Thus, we have shown the existence of one limit cycle for each value of c in the region $c_0 < c < c_D$.

If $c_0 > c_D$, similar arguments can be used to show that one unstable limit cycle exists for each value of c in the region $c_D < c < c_0$.

It is thus shown that the condition $c_0 = c_D$ is not necessary to describe the topology properly. As exposed by Jones et al., S_2 is in general not a center for $c = c_D$. This is a necessary condition for a center, but not at all a sufficient condition.

On similar reasons, Gunn's formula for v_D in the transfer-controlled case is in general invalid.

The apparent agreement between Gunn's formula and the domain velocity computed by Hauge, is partly due to the moderate doping levels ($n_0 = 10^{14}$ to 10^{15} cm^{-3}). By raising n_0 to 10^{16} cm^{-3} one obtains a greater deviation.

Einar J. Aas
Electronics Research Laboratory
Norwegian Institute of Technology
Trondheim, Norway
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