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A New Approach To Time-Domain Equalization*

Abstract: A theoretical study is made of the time-domain equalization procedures for the correction of delay distortion in high-speed data transmission lines. In the first part of the paper the conditions that insure valid and effective use of a class of conventional time-domain equalizers are reviewed. In the second part of the paper, a new type of nonlinear time-domain equalizer is proposed, in which iterative methods are not required. The theoretical basis is given for the new equalization method.

Introduction

In the time-domain method of correcting for the delay distortion of data pulses on telephone networks, the received pulse is operated upon with a series of delay lines. Automatic methods[1-3] for utilizing delay lines to correct the received waveform permit rapid adjustment of high-speed pulses on switched networks. The analyses in the present paper pertain to the efficiency of the equalization procedures.

The signal response of a transmission system to a given bit pattern is usually sampled at specified instants determined by the system clock. The information used for retrieving each bit pattern is completely contained in the received signal at these sampling instants. In this paper we will assume that the sampling instants are equidistant, which is almost always the case.

In the first part of the paper we discuss the conditions for efficient use of conventional linear devices for time-domain equalization and then we analyze the iterative processes. In the second part we describe a nonlinear equalizer that uses a decision threshold device. The proposed equalizer would have the decided advantage of operating without iteration processes. As an outgrowth of the analyses we show a hypothetical application of the nonlinear equalizer.

• *Signal-element response*

The signal-element response of the transmission system is the response of the system to a single bit, as indicated in Fig. 1. We shall represent this response by the polynomial $S(x)$:

$$S(x) = \sum_{i=-m}^{i=+n} s_i x^i \qquad n > 0 \quad m > 0, \tag{1}$$

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The quantity s_0 is the value at the sampling instant of the given bit. We shall refer to s_0 as the information sample. The s_i, where $i \neq 0$, are the values of the signal-element response at the sampling instants of the bits following and preceding the given bit. When the s_i for $i \neq 0$ are not all zero, the phenomenon commonly referred to as "inter-symbol interference" occurs.

The first nonzero received value is s_{-m} and the last one is s_n,

$$s_i = 0 \quad \text{for} \quad \begin{cases} i > n \\ i < -m \end{cases} . \tag{2}$$

The origin, $i = 0$, corresponds to the sampling of the given single bit. The value s_0 satisfies:

$$|s_0| \geq |s_i| \quad \text{for any } i. \tag{3}$$

In this paper we shall refer to systems in which the signal-element response for a ONE is $S(x)$ and the signal-element response for a ZERO is $-S(x)$.

• *Figure of merit for signal reception*

When a series of bits is sent, the m bits preceding a given bit and the n bits following it may interfere with this bit. For the worst-case pattern, the value V of the signal at the sampling instant for the given bit is

$$V = \epsilon(|s_0| - \sum_{i \neq 0} |s_i|), \tag{4}$$

where $s_0 = \epsilon\, s_0$

$\epsilon = \pm 1.$

In order to compare this value to the one corresponding to another signal, it is useful to define the figure of merit, $f[S(x)]$, denoted simply by $f(S)$:

$$f(S) = \frac{|s_0| - \sum_{i \neq 0} |s_i|}{|s_0|} \tag{5}$$

$$f(S) = 1 - \sum_{i \neq 0} \left| \frac{s_i}{s_0} \right|. \tag{6}$$

This figure of merit is such that:

If $f(S) \leq 0$, it is always possible to find a pattern such that at least one bit of this pattern will systematically be recovered in error.

If $f(S) > 0$, every bit of any pattern will theoretically be recognized without error in the absence of noise.

If $f(S) = 1$, the received signal is theoretically perfect for sampling. Maximum security in data transmission is expected (Nyquist's first criterion is satisfied).

Let us denote by $\hat{S}(x)$ an equalized signal-element response. The equalization capability of the equalizer will be measured by the quantity:

$$C = f(\hat{S}) - f(S). \tag{7}$$

For effective equalization C must be positive.

Utilization of conventional linear devices

• *Description*

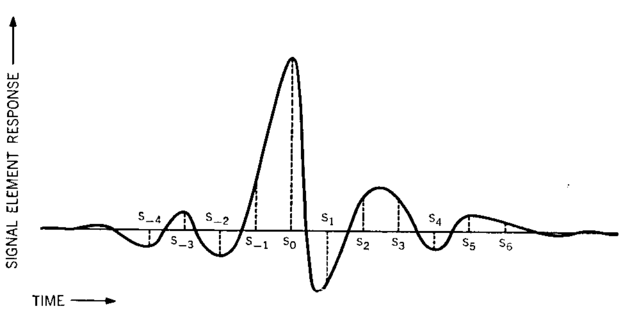
Conventional linear devices for time-domain equalization are described in Fig. 2. The symbol D represents an analog delay element. The delay of each of the $q + r$ delay elements is equal to the distance between two consecutive sampling times.

Let $P(x)$ be the polynomial defined by

$$P(x) = \sum_{i=-q}^{i=+r} p_i x^i \tag{8}$$

with $p_0 = 1$.

Figure 1 Signal-element response of a transmission system to a single bit before equalization.



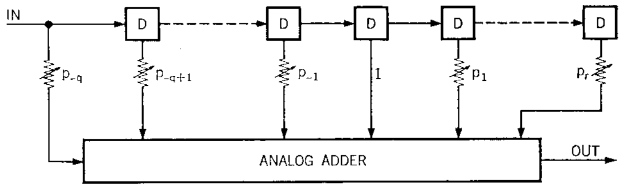


Figure 2 Conventional linear device for time-domain equalization. Units designated by D are analog delay elements.

If $S(x)$ is the signal entering this equalizer and $\hat{S}(x)$ is the signal coming out, we may write:[4]

$$\hat{S}(x) = S(x) \cdot P(x). \tag{9}$$

• *Purpose of equalization*

Perfect utilization of this conventional equalizer implies that the choice of the coefficients of the polynomial $P(x)$ yields a maximum value for $f(\hat{S})$.

An equalization procedure consists in defining a method for choosing the $q + r$ coefficients p_i, that is, a method for adjusting the $q + r$ weights, p_i, in the equalizer.

• *Conventional procedures*

In conventional procedures, one desires to choose the p_i such that[1,2]

$$\hat{s}_i = 0 \quad \text{for} \quad \begin{cases} 1 \leq i \leq r \\ -q \leq i \leq -1 \end{cases}. \tag{10}$$

The function $f(\hat{S})$ becomes in that case

$$f(\hat{S}) = 1 - \sum_{i<-q} \left| \frac{\hat{s}_i}{\hat{s}_0} \right| - \sum_{i>r} \left| \frac{\hat{s}_i}{\hat{s}_0} \right|. \tag{11}$$

One general way to proceed is to solve directly the linear system (10) of $q + r$ equations with $q + r$ unknowns.

Measurements of $f(\hat{S})$ will, then, indicate whether the distortion has been corrected or not, and in the case where it has been corrected, whether it has been sufficiently corrected or not. This solution is not very practical since it requires that either an analog or a digital computer be attached to the equalizer.

A second way to proceed is to choose a value α, to adjust p_α so as to cancel the signal at $i = \alpha$, and with an iterative process, to try converging to the solution of the linear system. Although there exist many other possible procedures (e.g., see Ref. 2) the study which follows is applicable to this second method.

As a matter of fact, what we are concerned with is not really the convergence to the solution of the linear system, but the certainty that this equalization iterative process is valid; in other words, we would like to be sure that an effective correction of the distorted signal will result from this equalization process.

A sufficient condition which insures that the equalization iterative process is valid for any α as pointed out by Lucky,[3] is

$$f(S) \geq 0. \tag{12}$$

- *Case when* $f(S)$ *is negative*

Conventional equalizers can still be efficient. But, in order to be sure of proper use of the equalizer, it is necessary to classify the set of all polynomials, $S(x)$, into several different families of polynomials and for each of them indicate the appropriate procedure. This, however, is not within the scope of this paper.

- *Example*

A particular family, for instance, is the one defined by:

$$S(x) = ax + 1 - ax^{-1}$$

$$n = m = 1$$

$$-s_{-1} = +s_1 = a$$

$$f(S) = 1 - 2\,|a|$$

If the equalizer is such that:

$$P(x) = -ax + 1 + ax^{-1}$$

$$\hat{S}(x) = P(x)S(x)$$

$$\hat{S}(x) = -a^2x^2 + (1 - 2a^2) - a^2x^{-2}$$

$$f(\hat{S}) = 1 - \frac{2a^2}{1 + 2a^2} = \frac{1}{1 + 2a^2} > 0.$$

For any a, $f(\hat{S})$ is positive. The equalizer, in that case, *is always efficient* ($C > 0$) and the procedure consists in directly adjusting p_{-1} and p_1 such that $p_{-1} = a$ and $p_1 = -a$.

- *Remark on the iteration process*

The gain, at each step of the procedure, may be very small, even if we are in the good case, $f(S) \geq 0$, and if the criteria which have been indicated are used. A large number of iterations may be required, and this number cannot be easily predicted (if it can be predicted at all) with conventional equalizers.

A new time-domain equalizer

The theoretical basis is given here for a new time-domain equalizer which has the great advantage of a simple procedure with no iterative process.

- *Equalization of the signal distortion preceding the information*

Description

The first part of the equalizer operates as the right half of a conventional time-domain equalizer, where $m - 1$ steps are used in the equalization procedure.

It is described in Fig. 3a. The number of delay elements is equal to $m - 1$ (the number of values, s_i different from zero, preceding the information, minus one).

Procedure

The j^{th} step consists in adjusting p_j so as to cancel the signal at

$$i = -(m - j).$$

This is essentially different from the conventional procedures mentioned previously:

$$p_j = -\frac{s_{-m+j}^{(j-1)}}{s_{-m}^{(j-1)}},$$

where s_I^J equals the value of sample I of the signal-element response after the J^{th} equalization iteration, and where

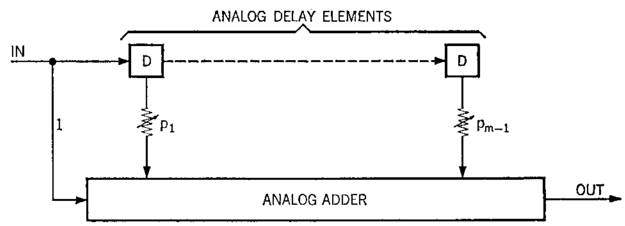
$$s_{-m+j}^{(j-1)} = s_{-m+j} + p_1 s_{-m+j-1} + \cdots + p_{j-1} s_{-m+1}.$$

Unlike conventional equalizers, p_j may be greater than 1.

The result is the following:

$$s_{-m}^{(j)} = s_{-m} \quad \text{for} \quad 1 \leq j \leq m - 1$$

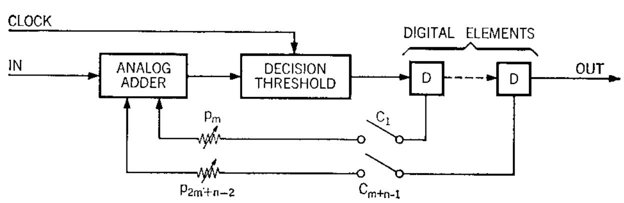
$$s_{-m+k}^{(j)} = 0 \quad \text{for} \quad \begin{matrix} 1 \leq j \leq m - 1 \\ 1 \leq k \leq j \end{matrix} \quad .$$



(a)

Figure 3a First part of proposed equalizer, operating as right half of conventional time-domain equalizer.

Figure 3b Second part of proposed equalizer utilizing nonlinear device.



(b)

After $(m - 1)$ steps the form of the signal-element response is such that:

$$S^{(m-1)}(x) = \sum_{i=0}^{i=n+m-1} s_i^{(m-1)} x^i + s_{-m} x^{-m}.$$

At this point, we have eliminated almost all signal distortion elements preceding the information sample.

Equalization of signal distortion preceding the information is not perfect, since s_{-m} cannot be canceled. Sometimes this equalization is made at the price of worsening the distortion following the information sample *and even including* the information sample itself. In fact, the information sample, after the first portion of the procedure, takes on the following value:

$$s_0^{(m-1)} = s_0 + s_{-1}p_1 + \cdots + s_{-m+1}p_{m-1}.$$

• *Equalization of the signal distortion following the information*

We shall suppose here that the information value $s_0^{(m-1)}$ has not been destroyed, that is,

$$|s_0^{(m-1)}| \gg |s_{-m}|, \quad (13)$$

and we shall denote by T the value

$$T = s_0^{(m-1)}/2. \quad (14)$$

Description

The second part of the equalizer consists in a nonlinear device[4] and is described in Fig. 3b. The number of the delay elements is $m + n - 1$. These elements are digital. Let us represent by 1 or -1, in binary data transmission, the information which is stored in these elements.

The initial state of each of the $m + n - 1$ switches is the open state (any initial pattern of 1 and -1 may be used in the delay element register). When the value of the signal s entering the decision threshold device is such that $|s| < |T|$ at the sampling time, the state of the switches will not be changed.

The role of the switches is to re-inject the signal when $|s| > |T|$. Under this condition,

1. if $s > 0$ the first digital delay element will contain 1;
   if $s < 0$ the first digital delay element will contain -1.
2. Switch C_1 will close. It will not be modified if it was already closed.

One sampling time later 1 or -1, corresponding to s, enters the second digital delay element,

1. 1 or -1, multiplied by p_m will enter the analog adder.
2. Switch C_2 will close and remain closed,

and so on.

Procedure

The m^{th} step in the procedure consists in adjusting p_{m+i} such that

$$p_{m+i} = -s_i^{(m-1)} \qquad 1 \le i \le m + n - 1.$$

This may be considered as only one step since the $m + n - 1$ adjustments may be accomplished simultaneously.

Result

In this equalizer, if we consider just a single signal element response, $s_0^{(m-1)}$ is used to cancel each value $s_i^{(m-1)}$ for $i > 0$, and s_{-m} is eliminated by the decision threshold device.

In a bit pattern response, the information sample value will be changed but all other distortion is eliminated in the absence of noise. In the presence of noise the effect of s_{-m} is to increase the probability of error somewhat.

Noise effect

The value of the incoming signal, at that level, is necessarily of the form $\pm K\,|s_0^{(m-1)}| \pm |s_{-m}|$

with $K = 0$ at the beginning of message reception

and $K = 1$ during all the time that information is being received (i.e., the complete message duration).

If the noise N is such that

$$|N| > |T| - |s_{-m}| = \frac{|s_0^{(m-1)}| - 2\,|s_{-m}|}{2}$$

an error may occur and generate a large burst of errors, but the fact that K must be equal to 1 during the whole time of message bit recovery can be taken into account, and errors can easily be detected. The complete evaluation of this first problem can be made only after the equalizer has been built and tested under standard line conditions. The device has not yet been built or tested.

• *Example of utilization*

We shall take a simple example where $S(x)$ is such that $m = 3, n = 4$.

$$S(x) = -0.1x^4 + 0.2x^3 + 0.3x^2 - 0.5x + 1 + 0.2x^{-1} - 0.1x^{-2} + 0.1x^{-3}$$

$$S'(x) = S(x) + xS(x) \qquad p_1 = 1, \qquad s'_{-2} = 0$$

$$S'(x) = -0.1x^5 + 0.1x^4 + 0.5x^3 - 0.2x^2 + 0.5x + 1.2 + 0.1x^{-1} + 0.1x^{-3}$$

$$S''(x) = S'(x) - s^2 S(x)$$

$$p_2 = -1, \qquad s''_{-1} = s''_{-2} = 0$$

$$S''(x) = 0.1x^6 - 0.3x^5 - 0.2x^4 + x^3 - 1.2x^2 + 0.3x + 1.3 + 0.1x^{-3}$$

$$|s_0^{(m-1)}| = 1.3 \gg 0.1 = |s_{-m}| \qquad T = 0.65.$$

It is interesting to note that the information sample value is no longer 1. The conventional equalizer has increased it to 1.3.

The p_i are proportional to $-0.1, +0.3, +0.2, -1, +1.2, -0.3$ for $i = 8, 7, 6, 5, 4$, and 3, respectively.

The configuration of this equalizer is given in Fig. 4.

Area of application of this type of equalizer

This example of a new time-domain equalizer utilizes $2m + n - 2$ delay elements:

$m - 1$	elements are analog delays
$m + n - 1$	elements are digital delays.

The $2m + n - 2$ equalization coefficients are adjusted in m different process steps. It can be used only if condition (13) is satisfied. This condition takes into account only the distorted values of the signal preceding the information but is still very strong. When this condition does not hold, this type of equalizer may still be quite efficient, but the exact configuration of the equalizer as well as the exact number of necessary steps in the procedure (which is still less than one iteration) are no longer systematically predetermined.

As soon as we have, at the j^{th} step,

$$|s_{-m+j+1}^{(j)}| \gg |s_{-m}|, \tag{15}$$

the value $s_{-m+j+1}^{(j)}$ may be used to cancel each value $s_i^{(j)}$ for $i > -m + j + 1$ with

$$T = \tfrac{1}{2} s_{-m+j+1}^{(j)}. \tag{16}$$

In the set of the analog delay elements j elements will, then, be used and in the set of the digital delay elements $2j + n$ elements will be used.

This possibility clearly extends the area of application of such equalizers, but we have not yet entered upon a systematic study of the signal class (or polynomial family) which will always be corrected.

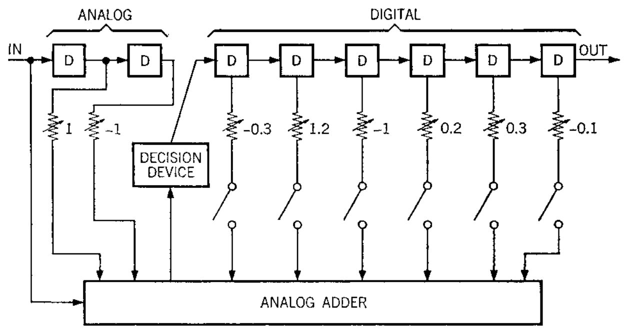


Figure 4 Configuration of equalizer for example given on page 231.

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