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Demagnetization of Flat Uniaxial Thin Films Under Hard Direction Drive

Abstract: The demagnetizing fields are derived for all points of flat uniaxial thin films under various drive fields. The derivation is accomplished by breaking up the flat film into a number of sheets in superposition and integrating their individual contributions to the demagnetizing fields. The scheme is self-consistent in that the magnetization results as a consequence of the derivation, and need not be assumed. Further, the accuracy does not depend on the position with respect to the edges, but rather on the number of sheets.

The general approach to the problem is discussed briefly and the final equation for a rectangular geometry given. The discussion is concerned with one-dimensional examples, demonstrating the somewhat unexpected form of the demagnetizing fields under various hard axis drive conditions. Single bits as well as continuous films of Permalloy driven by uniform fields and multiple strip lines are treated. The effect of registration on the demagnetization is also discussed.

Introduction

To properly understand the performance of a magnetic device, it is necessary to be able to describe in detail the magnetization in the magnetic medium. Certainly, we cannot hope to describe all the fields and variations in the magnetization; however, certain fields important to the dynamic operation of the device may be derived. One of these is the field resulting from the shape-influenced pole distribution, that is, the demagnetizing field.

While the solution for demagnetizing fields of ellipsoidal geometries may be found in the literature, the solution for other geometries is not well known.[1] It is often customary, in solving for the demagnetizing fields, to assume the magnetic medium to be uniformly magnetized, and to argue that the result in the central region will be reasonably accurate. The edges are expected to be very inaccurate. Some authors[2] reduce the edge inaccuracies by treating them separately. However, where the length-to-thickness ratio of the magnetized medium is small, serious errors may result in the central region as well.

In our method of solution, the magnetized medium is replaced by many coplanar thin sheets of different size and magnetization. The contributions from all the sheets are integrated to give the demagnetizing fields and hence the magnetization. The accuracy of the solution is seen to be dependent only upon the number of superposed sheets and the rate of change of magnetization through the point, and is independent of the relative position with respect to the edge of the magnetized medium. The solution is seen to be self-consistent in that the magnetization need not be assumed but results as a consequence of the derivation.

We will consider here a uniaxial thin film which has been initially set into one of the easy directions. It must be noted that the derivation is concerned only with the macroscopic "picture" of the magnetization and will not, for example, give any insight into the ripple[3] structure which may exist in the film. We ignore, then, any contribution due to stray fields[3] and exchange coupling, which may be shown to be negligible in our geometries.

Theory

A uniaxial thin film is generally operated by driving the magnetization into the hard direction by means of an applied field in that direction. When the field is relaxed, the magnetization will return to an easy direction under the influence of anisotropy forces. We will concern ourselves only with the demagnetizing field during the rotation into the hard direction, although the ensuing equations and results may be directly applied under other circumstances.

As the film is driven into a hard direction, variations will occur in the magnetization along both the hard direction and the easy direction. The poles resulting from the easy direction variation will appear at the corners and be of minimal consequence along the central hard axis. If we can be satisfied with the solution along the central hard axis, the contribution to the hard direction demagnetizing field of the poles resulting from the easy direction variation may be ignored. (For a consideration of the two-dimensional case, see Reference 5.)

The field, resulting from an elemental pole dm, and at some distance r from that pole, may in general be expressed as:

$$d\mathbf{H} = \frac{1}{r^2}\, dm\mathbf{r} \tag{1}$$

where $\mathbf{r}$ is the unit vector in the r direction.

Referring to Fig. 1a, the component of the demagnetizing field in the hard-direction, due to that component of magnetization directed in the hard direction, is given by

$$H_\delta = \int \frac{\cos\theta}{r^2}\, dm. \tag{2}$$

Assuming the sheet to be indefinitely thin and the magnetization to be uniform throughout,

$$dm = t\Delta M\, dx', \qquad x' = \frac{W}{2}x, \tag{3}$$

where ΔM is the difference in magnetization between the magnetized sheet and air and x is the decimal part of the half-width of the sheet, and

$$\cos\theta_1 = \frac{L/2(1-\rho)}{r_1}, \qquad \cos\theta_2 = \frac{L/2(1+\rho)}{r_2},$$

where ρ is the decimal part of the half-length of the sheet. Considering the poles at both ends, we have

$$H_\delta = \frac{tWL\Delta M}{4}\int_{-1}^{1}\left(\frac{1-\rho}{r_1^3} + \frac{1+\rho}{r_2^3}\right)dx \tag{4}$$

where

$$r_1^{\,2} = \left[\frac{L}{2}(1-\rho)\right]^2 + \left[\frac{W}{2}\right]^2,$$

$$r_2^{\,2} = \left[\frac{L}{2}(1+\rho)\right]^2 + \left[\frac{W}{2}\right]^2.$$

Now, let the flat film be made up of many coplanar flat sheets (Fig. 1) of varying size, each with uniform magnetization, in superposition, and identified by the index n. The reader will recognize that we are, in essence, using the familiar approach of "line charges" where the line charges are determined from the magnetization of the related sheet. The total demagnetizing field due to all the superposed sheets (or line charges) is then

$$H_D = \sum_n H_\delta, \qquad 1 < n < N, \tag{5}$$

where the solution to (4) is written with indices

$$H_\delta = \frac{2tW\Delta M_n}{L}\left[\frac{1}{\left(\frac{n}{N}-\rho\right)r_1} + \frac{1}{\left(\frac{n}{N}+\rho\right)r_2}\right], \tag{6}$$

and

$$r_1^{\,2} = \left[\frac{L}{2}\left(\frac{n}{N}-\rho\right)\right]^2 + \left[\frac{W}{2}\right]^2,$$

$$r_2^{\,2} = \left[\frac{L}{2}\left(\frac{n}{N}+\rho\right)\right]^2 + \left[\frac{W}{2}\right]^2.$$

For a uniaxial film,

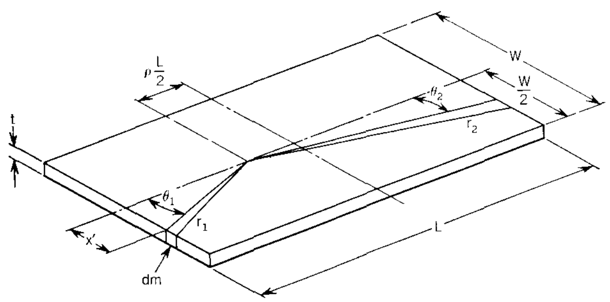
$$\Delta M_n = M_n - M_{n+1}$$

$$M_n = \frac{M_s}{H_K}\left[H_a(\rho) + H_D(\rho)\right] \quad \text{if} \quad H_a + H_D < H_K$$

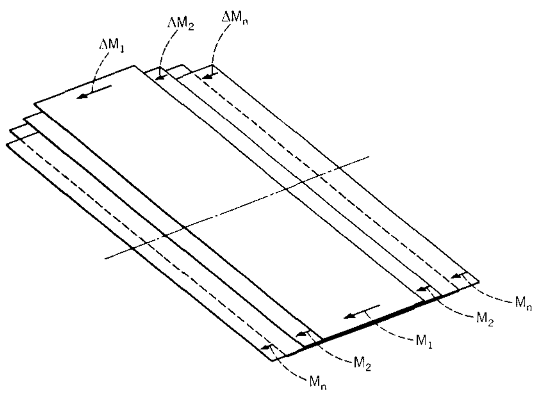
$$M_n = M_s \qquad\qquad \text{if} \quad H_a + H_D \geq H_K$$

where H_a is the applied field, H_K is the anisotropy field, and M_s is the saturation magnetization.

Figure 1 (a) Uniformly magnetized sheet (b) Superposed sheets.



(a)



(b)

We may, if we wish, express Eq. (6) succinctly in the form of an integral:

$$H_D(\rho) = \int_0^1 \{ K(\rho, \eta)\, \partial[M(\eta)]/\partial\eta \}\, d\eta \tag{7}$$

where K is easily deduced from (6) and $\eta \equiv n/N$.

It is immediately seen from Eqs. (6) and (7) that as the pole position, η, nears the observer position, ρ, a singularity exists in the expressions. However, it is evident (in the true physical circumstance) that such a singularity does not occur, since the sum of the components of the demagnetization factor cannot exceed 4π.

The singularity is removed by invoking the Cauchy Principal Value Theorem,

$$H_D(\rho) = P \int_0^1 [K(\rho, \eta)\, \partial M/\partial\eta]\, d\eta$$

where P indicates the principal value of the integral. If we consider the region within some arbitrarily small radius of the observer, the contribution from that region depends on the second derivative change in the magnetization through the region, since a contribution to H_D can occur only if the pole strength on one side of the singularity differs from that on the other side. Generally, the second derivative will be quite small, and the contribution to the demagnetization may be ignored; that is, the principal value tends to a small value.

The singularity is avoided in our solution by adopting an observer position between the line charges, and ignoring any contribution to the field that may have resulted from the material in that region.

Applications

Equation (6) was applied to typical problems involving rectangular flat-film geometries exhibiting a uniaxial anisotropy. Only the results of the computer solution will be discussed here. A discussion of the method of computation may be found in Reference 1.

We shall limit the applications to finite memory bits and continuous-sheet films which are driven by either a uniform field or a multiplicity of drive lines. Single-wire, two-wire, and three-wire drive schemes are considered, as well as a misregistration of a single-drive line. The drive line will be considered to be a current sheet of finite width and constant current density throughout its cross section. The field from a current sheet is obtained by integration of the contributions of elemental currents across the sheet,

$$H_a = \int_{-\lambda/2}^{\lambda/2} \frac{Ih\, dz}{2\pi\lambda\left\{\left[(\rho \pm b)\dfrac{L}{2} - z\right]^2 + h^2\right\}}, \tag{8}$$

where I is the total current of the sheet, λ is the width of

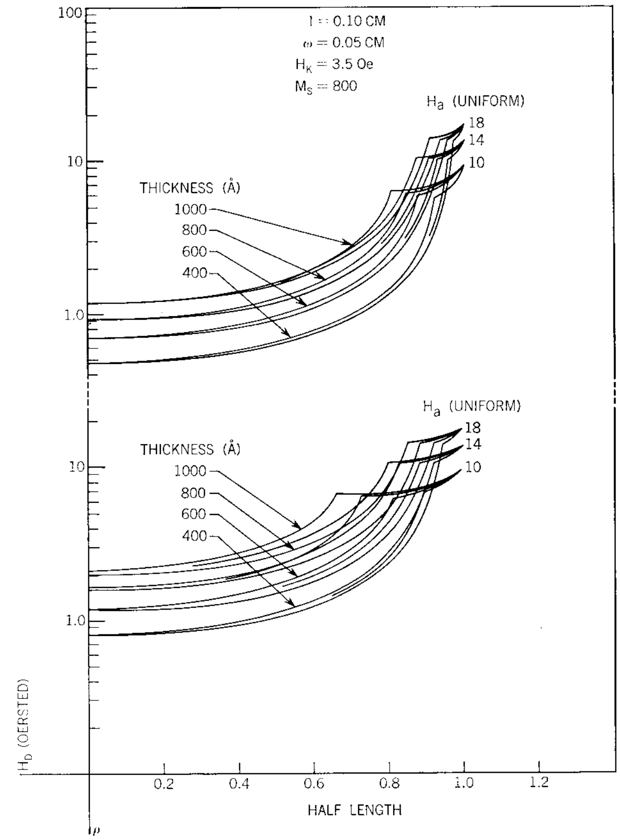


Figure 2 Uniform field drive.

the drive line, h is the distance of the line from the plane, ρ is the observer position in the plane, and b is the position of the drive line referenced to the observer position, $\rho = 0$. While the integral of Eq. (8) is easily obtained, it was expedient for computer solutions to leave it in integral form.

As a first example, typical memory bits are driven with a uniform applied field for comparison with bits driven by drive lines, Fig. 2. We note from the figures that the demagnetizing fields are strongly dependent on the geometry of the bit, but weakly dependent upon the magnitude of the field, except at the edges. This result is of course expected. Magnetization saturation is indicated by the discontinuity in the curves, and the film is found to be in saturation from the center of the bit to the position of the discontinuity. The demagnetizing fields for typical Permalloy films and bits are generally of the order of one oersted at the center of the bit.

Turning to drive lines, we find a marked departure from the uniform drive-field case, due to the rapid fall-off of the field toward the edges of the bit. This is most pronounced for a single drive line, as in Fig. 3a, where the drive field H_a is shown plotted along with the demagnetizing field.

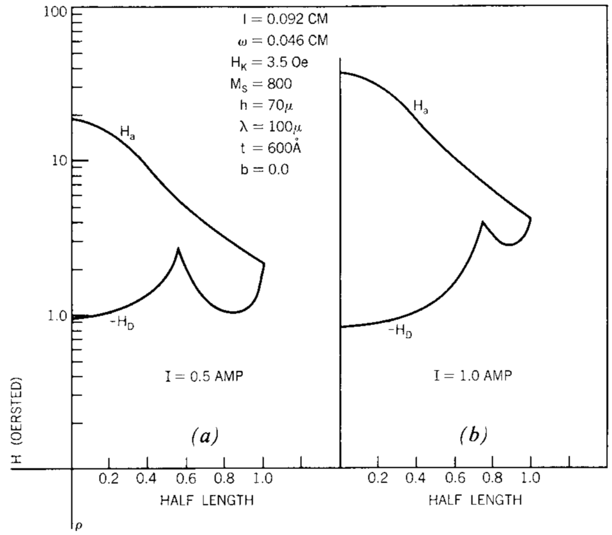


Figure 3 Single-wire drive.

For fields of 18 oersteds at the center of the bit, the magnetization is in saturation to only 50% of the bit, not because of demagnetization, but because of the fall-off of the field. We may extend the magnetization to 75% by doubling the current (Fig. 3b), but this is inefficient; the objective may be accomplished more effectively by using two drive lines with half the current, as shown in Fig. 4a. We may also increase the extent of magnetization by increasing the current per drive line (Fig. 4b).

If we space the drive lines farther apart (Fig. 4c) we find that the magnetization of the center drops below saturation with a marked drop in demagnetizing field, because of the formation of poles in the central region of the bit. We note further that strong demagnetizing fields occur in the central reaches of the bit as well as at the edges, which could result in reverse domains upon the removal of the hard direction drive. Figure 4d indicates that the demagnetizing field is not appreciably altered by a moderate change in geometry. Spacing the drive lines farther apart (Fig. 4e) worsens the situation, but this may be easily corrected, as expected, by increasing the current in the drive lines (Fig. 4f).

If two drive lines are better than one, then three should be better than two. That this is the case is shown in Fig. 5. The central drive line prevents the formation of poles in the central region of the bit. Moderate changes in geometry do not materially alter the magnetization (Figs. 5b and 5c), but increasing the current in the drive lines extends the saturation nearly to the edges.

It will be of interest to investigate the demagnetizing fields occurring when a continuous film is driven into its

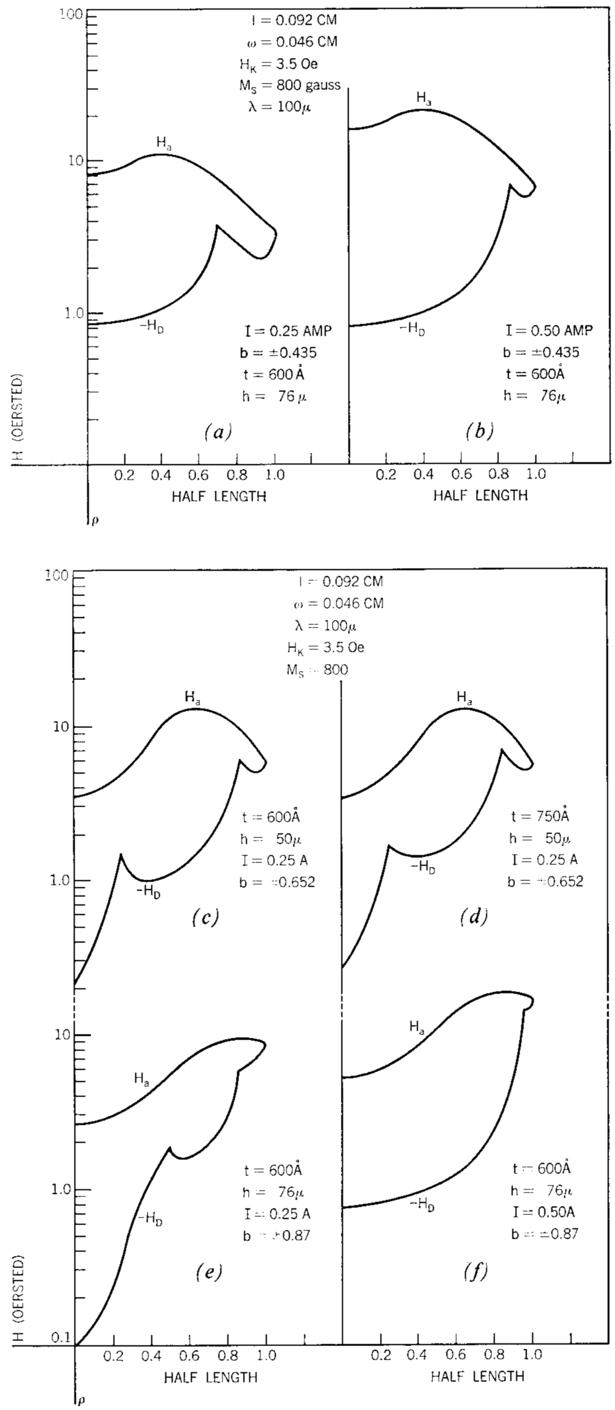


Figure 4 Two-wire drive.

hard direction by drive lines. To correlate the results with the discrete bits previously discussed, the dimensions are normalized against the bit dimensions. While one should investigate continuous films using a variety of drive lines, we shall consider the case with two drive lines,

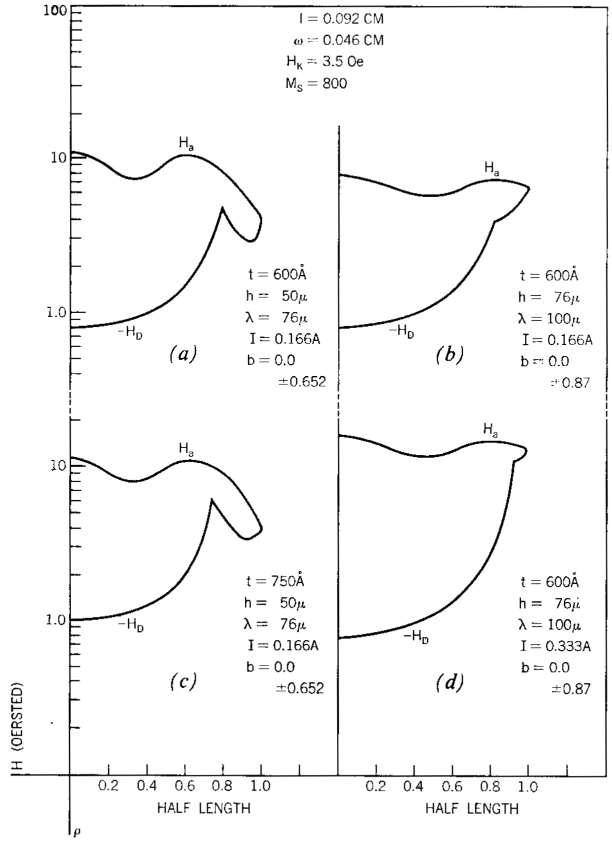


Figure 5 Three-wire drive.

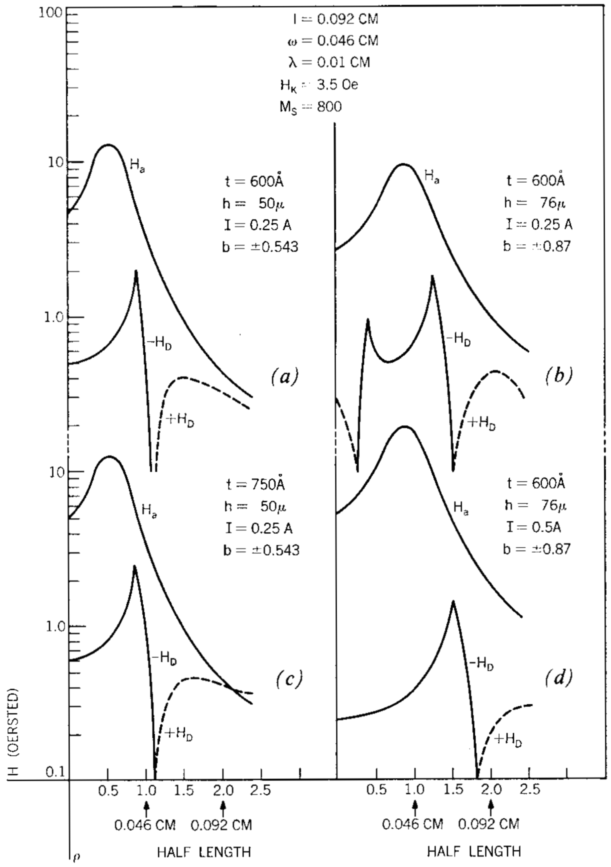
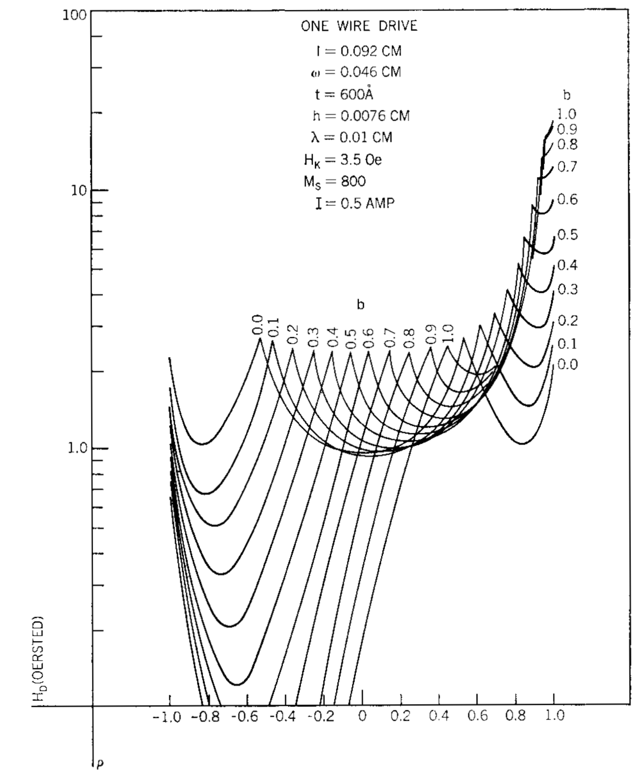


Figure 6 Two-wire drive; continuous film.

Figure 7 Registration.



since it is likely to give the least predictable results. We see immediately from Fig. 6a that the demagnetizing field changes sign as one proceeds from the central region of the bit. This change in sign results in a larger bit size than expected from the drive field alone. If we now weaken the field in the center of the bit (Fig. 6b), we see that the demagnetizing field in the central region also changes sign and becomes positive, aiding in the magnetization of that region. A comparison of Figs. 6a and 6c indicates that moderate changes in geometry do not materially alter the demagnetizing fields. It is found, however, that changes in drive-line current (Figs. 6b and 6d) result in a markedly changed bit size. It is quite evident that control of bit size in continuous films will depend upon the field gradient of the drive lines.

As a final point of interest, we consider the effect of misregistration of a single-drive line on the demagnetizing field for the case of discrete bits (Fig. 7). The demagnetizing field is seen to drop off rapidly on one side and increase on the other with a radical decrease in bit size (magnetization saturation). The effects of misregistration are in evidence with the onset of displacement of the drive line.

Summary

The demagnetizing field of a flat uniaxial thin-film memory bit has been derived for various drive configurations by breaking the bit into a large number of sheets in superposition and integrating their individual contributions to the field. It is seen that no *a priori* knowledge of the magnetization or demagnetizing field is necessary and that the magnetization is obtained as a consequence of the calculation. The method of derivation is therefore said to be self-consistent.

By studying both discrete bits and continuous sheets, we find that the demagnetizing field is determined by the distribution of poles and, therefore, is a consequence of both the physical geometry of the film and the field geometry of the drive lines. That is to say, the demagnetizing field cannot be determined from the film geometry alone. It is improper to speak of a demagnetization factor for a film geometry; demagnetization is a point concept and no single value may be assigned to a particular geometry except in the case of ellipsoids of revolution.

While we usually think of demagnetizing fields as being oppositely oriented to the magnetization (hence demagnetizing), we find that unique conditions may arise where the fields are in fact oriented in the same direction as the magnetization. These conditions exist where the bit is very much larger in size than the actual area driven by the drive lines and, again, where the fields at the central region of the bit are lower in amplitude than those in the adjacent regions.

Calculations developed by the method of superposed sheets represent a second-order approximation and are by no means exact. We assume a one-dimensional model and are satisfied to obtain a solution along the central hard axis. The accuracy of this model depends upon the number of superposed sheets chosen (in this case, 41) rather than upon the proximity of the observer position to the edges of the film.

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