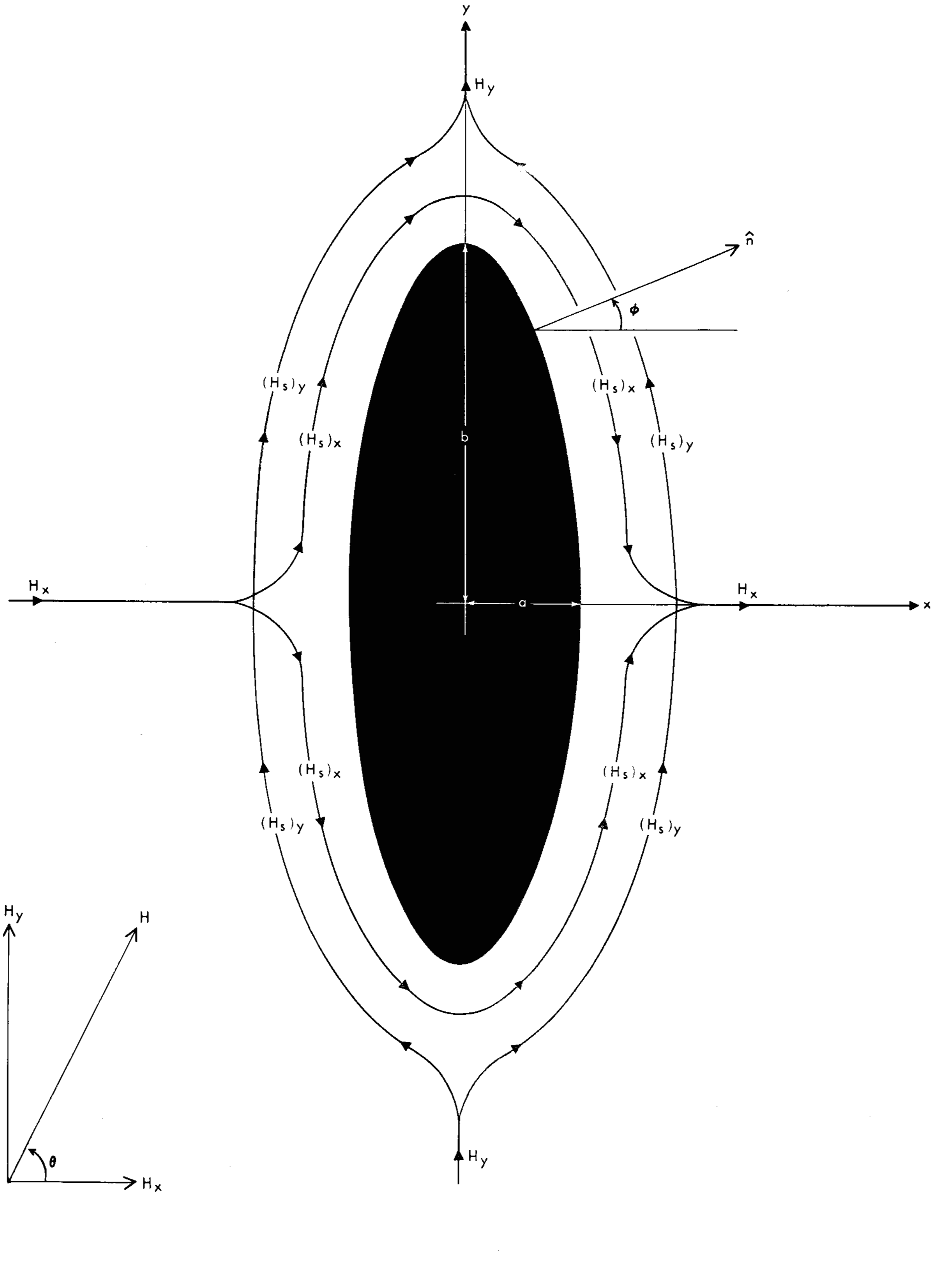


Figure 1 **Surface magnetic fields around a superconducting ellipsoidal cylinder.**

M. D. Reeber

Geometric Effects in the Superconducting Transition of Thin Films

Abstract: A study is made of the effect of geometric factors and field orientation on the superconducting transition of bulk material whose dimensional ratios are comparable to those of thin evaporated films. An expression is derived for an effective demagnetizing coefficient in the direction of the applied field for an elliptic cylindrical superconductor, and it is shown that this deviates from 1 by quantities of the order of 10^{-2} or smaller for experimentally realizable conditions with typical films. These small coefficients give rise to surface fields sufficiently large for transition to the intermediate state. The shape of the transition for various field orientations has been obtained experimentally, and these curves are analyzed qualitatively on the basis of Landau's and Andrew's theory of the intermediate state. For temperatures below the transition point, and in the absence of fields other than the earth's magnetic field, the possibility that these films are in the intermediate state is explored.

The use of thin superconducting films as computer elements requires some knowledge of the manner in which resistance will be restored to a segment of film by the application of an external magnetic field. Since a film may be typically several thousandths of an inch in width and perhaps a thousand Angstroms thick, it will produce severe distortions in any component of the magnetic field normal to the plane of the film. The following work was undertaken to gain information about the manner in which resistance might be expected to appear in a film subjected to a variety of applied magnetic fields. The experimental work was done with foils instead of films in order to provide specimens having uniform internal structure and geometry.

It was first shown experimentally by Meissner and Ochsenfeld[1] that a pure superconductor will exclude magnetic flux from its interior. This exclusion of flux in the superconductor leads to a surface field, H_s, that is larger than the applied magnetic field, H_a. For an ellipsoidal superconductor with an external magnetic field parallel to a principal axis, H_s can be expressed as[2]

$$H_s = H_a \sin\phi'/(1-n), \qquad (1)$$

where ϕ' is the angle between the normal to the surface and the direction of H_a, and $4\pi n$ is the demagnetizing coefficient in the direction of H_a. At $\phi'=\pi/2$, H_s achieves its maximum value:

$$H_{s\,\max} = H_a/(1-n). \qquad (2)$$

The demagnetizing coefficient is a purely geometric factor, depending only on the axial ratios of the ellipsoid, and its value has been calculated for the general ellipsoid and various limiting cases.[3,4,5]

As H_a is increased, the superconductor will continue to exclude the magnetic flux until $H_{s\,\max}=H_c$, the critical field of the superconductor. If a specimen is large enough so that surface energy is small compared to volume energy, then for values of $H_a>(1-n)H_c$ the specimen is in a macroscopically uniform intermediate state with the magnetic induction B described by[2]

$$B = H_c - (H_c - H_a)/n\,.$$

The volume fraction of material normal, f_n, is defined as B/H_c. If the specimen is small enough so that surface energies cannot be neglected, the entrance into the intermediate state is delayed until $H_{s\,\max}/H_c=\rho$, where ρ is some number larger than $(1-n)$. In general, $\rho=(1-n)+f(\Delta'/L)$, where $\Delta'=(8\pi\alpha/H_c^2)-\lambda$, L = length of the superconductor in the direction of the applied field, α = surface energy between normal and superconducting region, and λ = penetration depth of the field into the superconductor. The precise form of the function f is in doubt[6,7,8] but it is generally agreed that it increases monotonically with increasing Δ'/L.

Theory

We shall now obtain the effective demagnetization factors for an elliptic cylindrical superconductor which should approximate the conditions obtained with a thin film. Consider the elliptic cylindrical superconductor oriented so that its major axis is parallel to the z axis (Fig. 1). Let the x and y axis be the minor and intermediate axes, respectively. Let the applied fields be in the x-y plane at an angle θ to the x-axis so that $H_x = H_a \cos\theta$, and $H_y = H_a \sin\theta$.

At the surface of the superconductor, the magnetic field is tangential to the surface since B is continuous across the boundary; $B=0$ inside, and $B=H$ outside. The magnetic field at the surface is the linear sum of the surface fields due to H_x and H_y. We can write, using Eq. (1),

$$H_s = \frac{H_a \cos\theta \sin\phi}{1-n_x} + \frac{H_a \sin\theta \cos\phi}{1-n_y}, \qquad (3)$$

where[3] $n_x = 1/(1+a/b)$, (4)

$$n_y = 1/(1+b/a), \qquad (5)$$

in which a is the length of the minor axis, b is the length of the intermediate axis, and ϕ is the angle between the normal to the surface and the x-axis. The value of ϕ for $H_{s\,\max}$ can be obtained by differentiation of Eq. (3) which yields

$$\cot\phi_{\max} = (a \tan\theta)/b\,. \qquad (6)$$

Substituting Eqs. (4), (5), and (6) into Eq. (3) yields

$$H_{s\,\max} = (1/ab)[H_a(a+b)(b^2\cos^2\theta + a^2\sin^2\theta)^{1/2}]\,.$$

An effective demagnetizing coefficient, $4\pi n_{\text{eff}}$, can be defined by analogy with Eq. (2), i.e.,

$$1-n_{\text{eff}} = H_a/H_{s\,\max}\,,$$

which for this case leads to

$$1-n_{\text{eff}} = ab/[(a+b)(b^2\cos^2\theta + a^2\sin^2\theta)^{1/2}]\,. \qquad (7)$$

For thin superconducting films the ratio of a to b may easily be of the order of 10^{-4} or smaller. In this case Eq. (7) may be simplified to read

$$1-n_{\text{eff}} = a/(b\cos\theta), \qquad (8)$$

for $\theta \neq \pi/2$. For $a/b = 10^{-4}$, Table 1 shows $1-n_{\text{eff}}$ for some chosen values of θ near $\pi/2$. Thus under normal experimental conditions, where the alignment of the specimen and the applied field is no better than 30′, n_{eff} differs from 1 by quantities of the order of 10^{-2} or smaller.

Table 1. **Effective Demagnetizing Coefficient for θ Near 90°.**

θ	$1-n_{\text{eff}}$
89°59′	3×10^{-1}
89°30′	1×10^{-2}
89°	6×10^{-3}
88°	3×10^{-3}
87°	2×10^{-3}

Thus, for the dimensions used in the example, if the applied field, including the earth's field, is not parallel to the major axis of the specimen to within 30′, then at some point on the surface of the specimen there will be a field at least 100 times larger than the total applied field. A field of 10 oersteds misaligned 30′ will give rise to a field of 1,000 oersteds at some point on the surface of the specimen. The same field of 10 oersteds misaligned 2° will give rise to a field >3,000 oersteds at some point on the surface of the specimen.

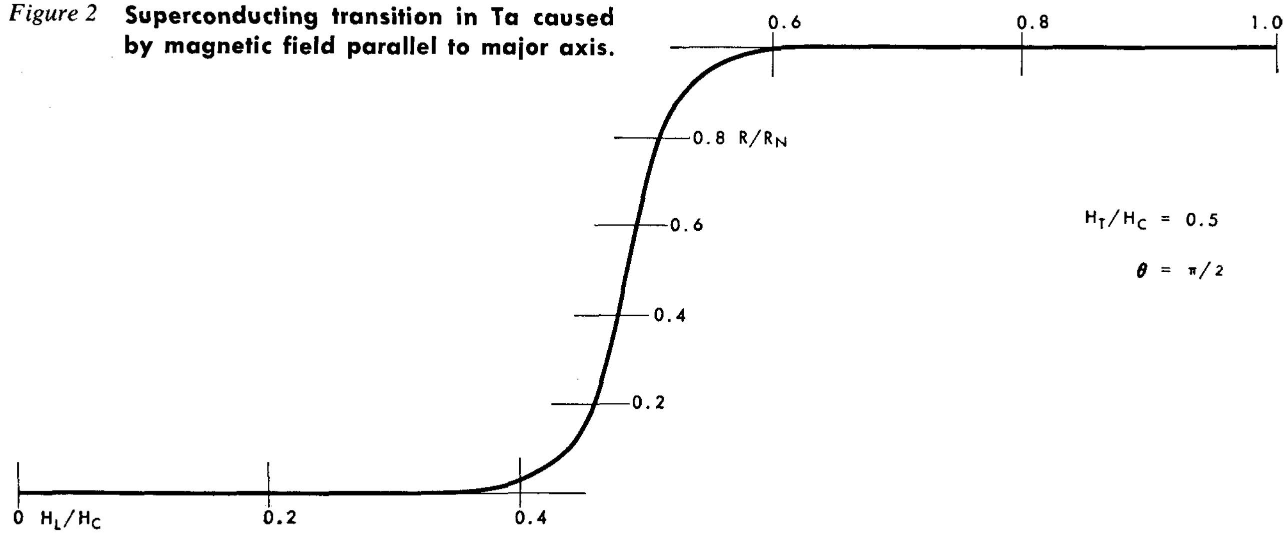
Taking another point of view, if there exists a component of field parallel to the minor axis of the specimen, this component of field, multiplied by 10^4, will appear somewhere on the surface of the specimen. A stray field of 1/2 oersted in this direction will give rise to a surface field of 5,000 oersteds. In many cases this surface field will be large enough to cause a transition into the intermediate state.

Experimental Details

Since there are numerous difficulties in the fabrication and measurement of thin homogeneous films, experiments were carried out using thin foils.

The specimen used was a rectangular strip of 0.25-mil tantalum, 0.125 inch by 1.25 inches, with axial ratios of 1:500:5000. The specimen was initially purified by intermittent heating *in vacuo* at 2200°C until the system pres-

Figure 2 **Superconducting transition in Ta caused by magnetic field parallel to major axis.**



sure with the specimen heated was less than 5×10^{-8} mm of Hg, as read on an ionization gauge. The ratio of the specimen resistance at liquid-helium temperature to that at room temperature was 1:250, indicating a relatively high degree of purity for tantalum. The specimen was mounted on a rotary shaft positioned inside a Helmholtz coil made of niobium wire, which was fixed in the helium dewar flask so that its field was perpendicular to the long dimension of the strip. This coil supplied a transverse field uniform to within 2% over the length of the sample. A solenoid with overwound ends according to a design recommended by Garrett[9] was mounted in the surrounding liquid nitrogen and supplied a longitudinal field uniform to one part in one thousand over the volume occupied by the sample.

Measurements of the transition were made by reading the voltage between two probes spaced 0.5 inch apart, with a constant current through the specimen.

Results

The experimental results are best summarized by means of Figs. 2 to 5, which show the variation of resistance with external magnetic field for a number of orientations. All resistance values have been normalized to the resistance of the specimen in the normal state, and all magnetic field values have been normalized to the magnetic field

Figure 3 **The change in longitudinal magnetic field required for a transition with a fixed transverse magnetic field at $\theta=0$.**

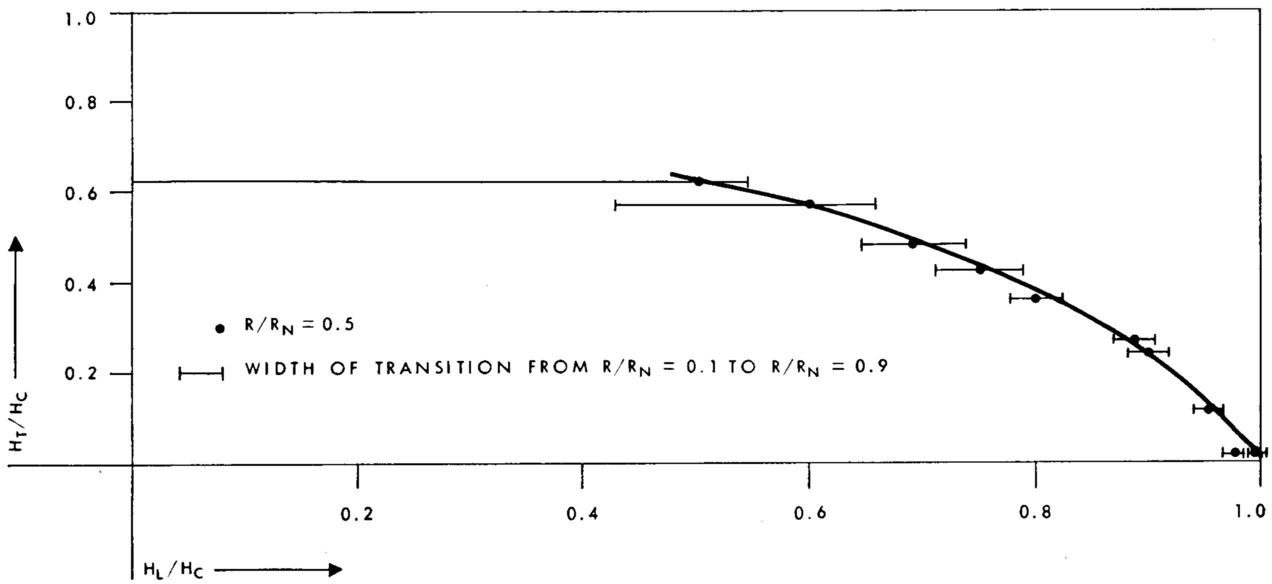
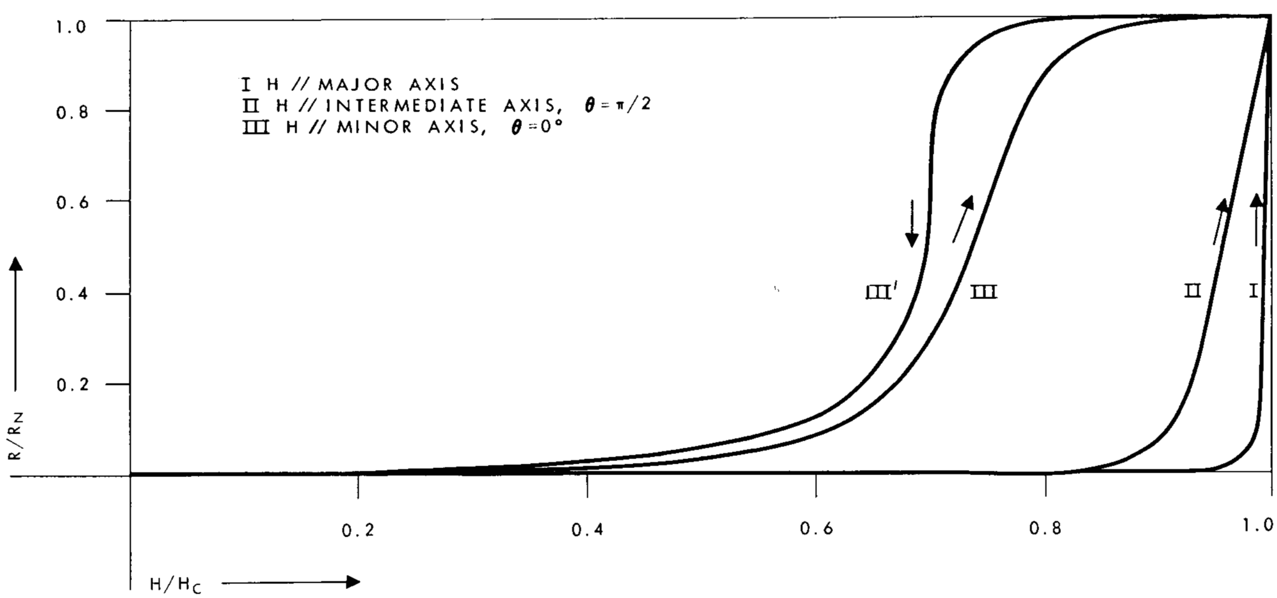


Figure 4 **Superconducting transitions obtained with magnetic fields parallel to principal axes.**



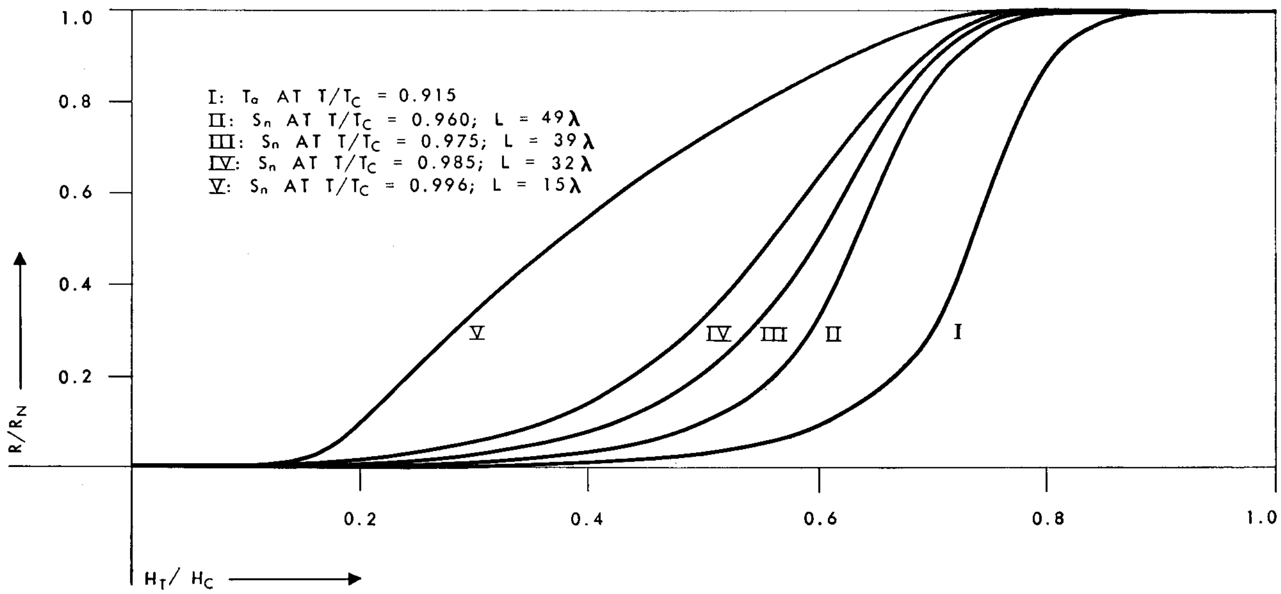


Figure 5 **Superconducting transitions obtained with magnetic field at θ=0.**

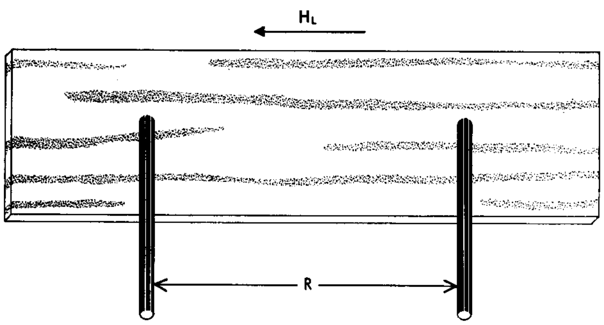


Figure 6 **Possible intermediate-state pattern for magnetic field parallel to major axis.**

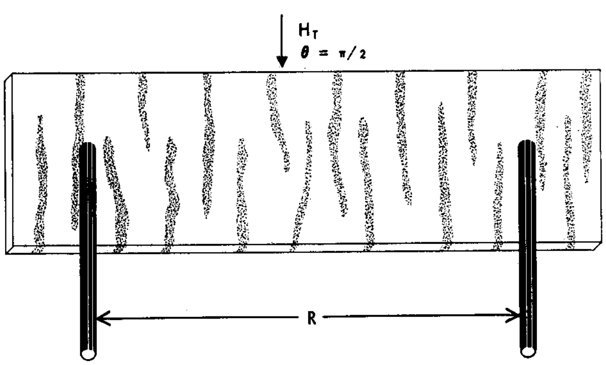


Figure 7 **Possible intermediate-state pattern for a magnetic field parallel to intermediate axis.**

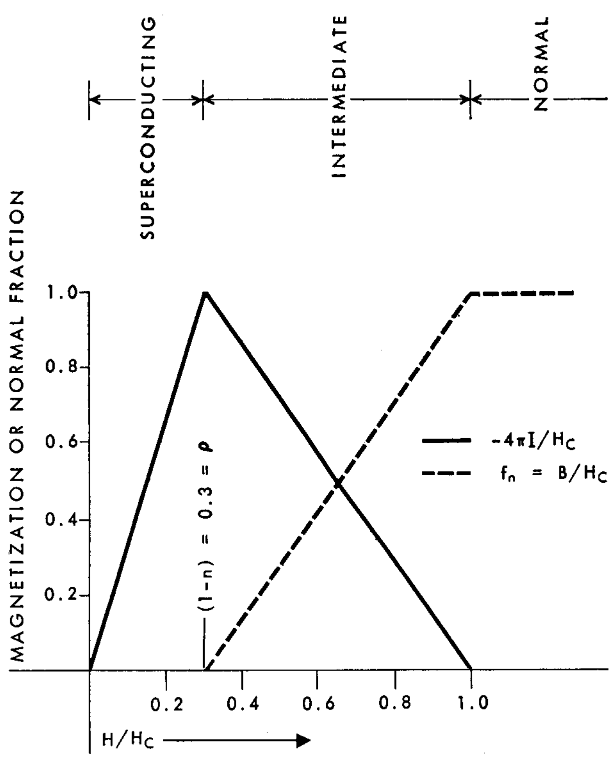


Figure 8 **Magnetization and normal fraction curves for a large specimen, neglecting surface energy.**

along the major axis of the specimen required for the transition from the superconducting to the normal state. The angle θ is the same as defined in Fig. 1. H_L is the applied field along the major axis of the specimen, (i.e., the field supplied by the external solenoid) and H_T is the applied field at angle θ (i.e., the field supplied by the Helmholtz coil). Fig. 2 shows a typical plot of a transition obtained by increasing H_L while keeping H_T constant at $\theta = 0$. Fig. 3 shows a shift in H_L, with a fixed applied H_T at $\theta = 0$. Fig. 4 shows the transition as measured with applied fields in the direction of the principal axis of the specimen. Fig. 5 shows the transition as obtained by varying H_T at $\theta = 0°$ for various materials at different temperatures. Measurements on tin showed the same characteristic behavior as those on tantalum.

Discussion

In the following discussion we assume that the magnetic behavior of a bulk specimen is essentially that of its inscribed ellipsoid. As already pointed out, n_{eff} is approximately 1 for all field orientations of the specimen. If we examine the transition for various orientations, Fig. 4, we find a variety of curves despite the fact that the demagnetizing coefficient has not changed appreciably. For the fields parallel to the long and intermediate axes, $\rho \approx 0$ since $1-n$ is 10^{-2} or less and $f(\Delta'/L)$ is negligible. If we assume an intermediate-state pattern consisting of regions whose boundaries are approximately parallel to the applied field, we would expect the longitudinal transitions to correspond to those actually observed. As shown in Fig. 6 for the case where H is parallel to the long axis, the superconducting regions will short-circuit the normal regions until, at a value of the applied field close to the critical field, the superconducting regions disappear completely.

In Fig. 7, for the case where H is parallel to the intermediate axis, a superconducting path can exist for values of H much greater than that necessary to start the intermediate state, but as the applied field is increased the normal regions grow until they form a complete strip across the specimen. In this case, resistance will appear at lower fields than in the case of the longitudinal field. The specimen will not show full normal resistance, however, until $H = H_c$ and all superconducting regions have vanished.

If we now consider the case where the applied field is parallel to the short axis of the specimen we note that $f(\Delta'/L)$, while still small, is indeed finite. Landau's theory of the intermediate state[6] predicts qualitatively a delay of the entrance into the intermediate state, and the attainment of a pure superconducting state at fields lower than the critical field H_c. If we examine a plot of the magnetization of the specimen versus the applied field, Fig. 8, we see that, in the case where we neglect surface energy effects, the magnetization rises uniformly until at $H/H_c = (1-n)$ the specimen enters the intermediate state. The initial rise in magnetization is due to the flux-excluding properties of the superconductor and the slope of the curve is $1/(1-n)$. The area under the curve represents

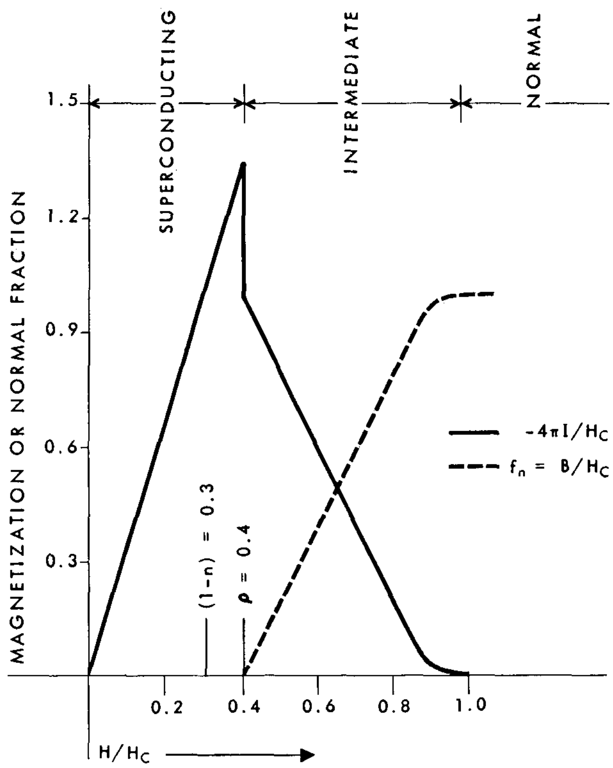


Figure 9 **Magnetization and normal fraction curves for small specimen, including surface energy.**

the total increase of energy resulting from the transition to the normal state, i.e., $H_c^2/8\pi$ per unit volume. When the surface energy is large the entrance into the intermediate state is delayed by an amount $f(\Delta'/L)$. For the case where λ is small compared to the dimensions of the specimen, the magnetization is increasing uniformly at the rate of $1/(1-n)$ as H is increasing. In order that the area under the magnetization curve be equal to $H_c^2/8\pi$ per unit volume, it is necessary that the normal state be almost completely attained at $H < H_c$. The maximum value that $f(\Delta'/L)$ can have is that value of $f(\Delta'/L)$ for which $\int^{H_c\rho_{\max}} I\,dH = -H_c^2/8\pi$. Integrating this expression, using $-4\pi I = (1/1-n)H$, yields $\rho_{\max} = \sqrt{1-n}$. This corresponds to a normal fraction curve as shown in Figs. 9 and 10. It is easy to visualize how a random distribution of normal and superconducting regions such as that shown in Fig. 11 can lead to the curve obtained experimentally (Fig. 4, Curve III).

As we increase the temperature toward T_c the penetration depth of the magnetic field into the superconducting regions increases. The slope of the magnetization curve must decrease to some value $< 1/(1-n)$, allowing ρ to increase beyond $\rho_{\max}$, leading to the shift in transition observed in Fig. 5. Note that for curve V, Fig. 5, the slope has decreased to the extent that ρ is appreciably different from 0 and the curve obtained is qualitatively similar to those obtained in the experiments of Andrew[7] in the case of cylinders in transverse fields.

It may also be pointed out that the transition observed

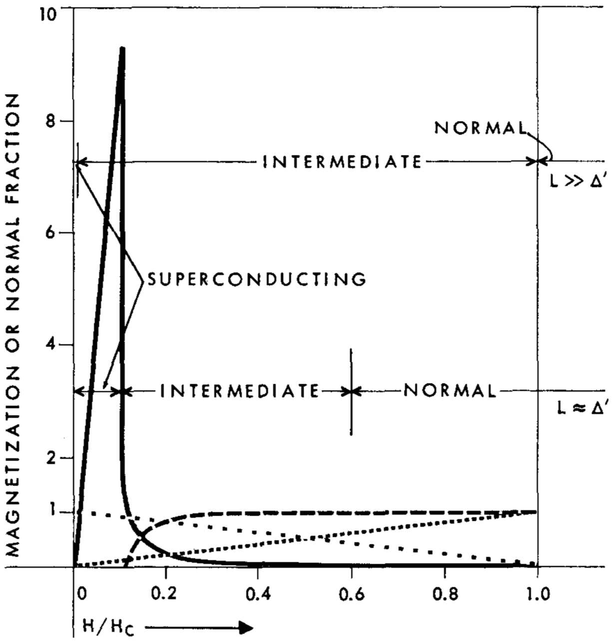


Figure 10 **Magnetization and normal fraction curves for specimen with $(1-n)=0.01$.**

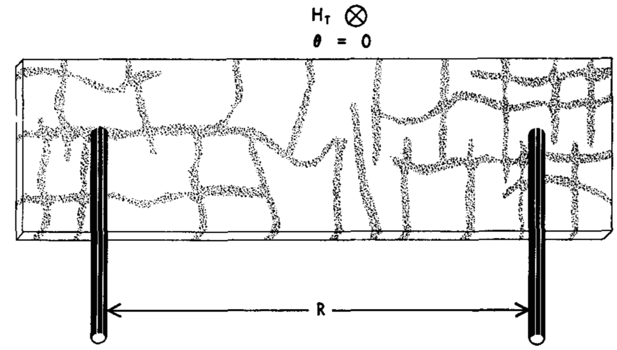


Figure 11 **Possible intermediate-state pattern for a magnetic field parallel to minor axis.**

by decreasing H_L from some value above H_c (Curve III', Fig. 4) leads to "supercooling" effects of precisely the same form as those observed by Andrew.[7]

Conclusions

On the basis of the foregoing, the following conclusions about the effect of geometry on the transitions of thin films can be made:

1. The demagnetizing coefficient, $4\pi n_{\text{eff}}$, as a function of the direction of the applied field, can be expressed as $1-n_{\text{eff}}=ab/(a+b)(b^2\cos^2\theta+a^2\sin^2\theta)^{1/2}$. For most typical films, this reduces to $1-n_{\text{eff}}=a/b\cos\theta$.
2. Under normal experimental conditions, i.e., field alignment no better than $\pm 1/2°$ and for films with axial ratios of 10^{-4} or smaller, $1-n_{\text{eff}}<10^{-2}$.
3. For cases where $L>\lambda$, $1-n_{\text{eff}}<\rho<(1-n_{\text{eff}})^{1/2}$, and the intermediate state forms under the application of very small magnetic fields. (In most instances the earth's field is large enough to induce the intermediate state.)
4. When $L<\lambda$, ρ can increase beyond $\sqrt{1-n_{\text{eff}}}$. This will happen if the film is extremely small, the temperature is extremely close to T_c, and the transition is caused by a transverse field at $\theta\approx 0°$.
5. The shape of the transition curves agrees with qualitative aspects of the various theories, but a quantitative comparison with any particular theory can not be obtained because of the smallness of ρ.

Acknowledgments

The author wishes to express his gratitude to Dr. William B. Ittner III for his invaluable assistance in the initial conception and early stages of this project, and to F. Robert Hand for the use of his data on tin specimens.

List of mathematical symbols

a = length of minor axis
b = length of intermediate axis
H_a = the magnetic field that would exist if no superconductors were present
H_c = the critical magnetic field of the superconductor
H_s = the magnetic field on the surface of the superconductor
H_T = H_a parallel to the minor axis of the superconductor if $\theta=0$, or
= H_a parallel to the intermediate axis of the superconductor if $\theta=\pi/2$
H_L = H_a parallel to the major axis of the superconductor
L = the length of the superconductor in the direction of H_a
$4\pi n$ = demagnetizing coefficient
α = surface energy per unit area between normal and superconducting region
λ = penetration depth of magnetic field into the superconductor
θ = angle between the applied field and the x axis
ϕ = angle between the normal to the surface and the x axis
ϕ' = angle between the normal to the surface and H_a

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