P. R. Strickland

The Thermal Equivalent Circuit of a Transistor

Abstract: An exact electrical analogue is given for the thermal system between the collector junction and the constant-temperature environment of a transistor. For this circuit analogue, the voltage response to an applied current is equivalent to the temperature response of the collector junction to an applied-power dissipation. The objective of this paper is (1) to prove that this thermal equivalent circuit is entirely consistent with the rigorous, academic approach to the problem, which is to solve a boundary-value problem for heat flow in the transistor system; (2) to present an experimental method for obtaining the circuit parameters in the thermal equivalent circuit; and (3) to demonstrate the utility of the thermal equivalent circuit for the circuit designer and the transistor designer.

Introduction

This paper discusses a practical method for determining the operating temperature of a transistor collector junction. The temperature of this minute region is an important consideration because (1) many of the electrical characteristics of a junction are temperature dependent, (2) there appears to be a maximum permissible junction temperature for any desired life expectancy of the transistor, and (3) the power-dissipation rating of the transistor can be improved if it can be designed to reduce the large thermal gradients between the collector junction and the environment. It is evident that the temperature at the junction will be related in some way to the electric power applied to the transistor and the ability of the transistor materials to dissipate heat energy. The problem, then, is to establish the function which relates junction temperature to the thermal system and the power dissipation. This paper gives an electrical analogue in a network composed of lumped *RC* elements. The voltage at the two terminals then corresponds to the collector-junction temperature.

In the past much emphasis has been placed on a quantity called the *K-factor*, or thermal resistance, of a transistor. This *K*-factor is obtained experimentally from the ratio between the value of the *constant* temperature rise of the collector junction and the *constant* value of power dissipation which produced this rise. *K*-factor, then, is a measure of the power-dissipating ability of the device and also can be used to determine the junction temperature for a known constant-power dissipation. In practice, however, this *K*-factor is often multiplied by an *average* power dissipation, and the result is assumed to be the *average* junction temperature. But the *K*-factor alone, for instance, could not specify the period of time a large power pulse of known amplitude can be applied to a device before its maximum junction temperature is exceeded. It is evident, then, that the *K*-factor of a device is not adequate to indicate temperature variations in a transistor resulting from variations in the applied power.

An approach to this problem, which is based on solid theoretical ground, is to solve the boundary-value problem posed by the partial-differential equations for heat flow. One example of this approach successfully applied to the geometry of the rate-grown transistor is reported by Mortenson.[1] The difficulties with this method are (1) that to formulate the problem requires a great deal of knowledge about the internal geometry of the device, (2) that certain physical constants, such as the heat conductivity, diffusivity, and heat-transfer coefficients must be available for every material in the device, and (3) that the solutions of certain equations which are often needed can be obtained only by the methods of numerical analysis. These problems may be solved by making simplifying assumptions, but the results are then dependent on the validity of the assumptions, and hence, on the ability and judgment of the problem-solver.

A third approach, which is an extension of the *K*-factor concept and also consistent with the boundary-value approach, is to propose a thermal equivalent circuit for the transistor. The circuit representation of a thermal system is permitted by an analogy between certain electrical and thermal quantities. These analogous quantities are listed in Table I. An example of a quantitative attempt to use this thermal-equivalent-circuit approach was reported by Simons,[2] although many others have suggested its use in a qualitative manner. Apparently no one has shown that

Table I **A list of analogous quantities between thermal and electrical systems.**

Electrical		Thermal	
V	voltage (volts)	T	temperature (°C)
I	current (amps)	P	power dissipation (w)
R	electrical resistance (ohms)	R	thermal resistance (°C/w)
C	electrical capacitance (farads)	C	thermal capacitance (w-sec/°C)

the quantitative use of this circuit is justified or that it is as valid as the rigorous, academic approach mentioned above. In addition, the particular thermal equivalent circuit which has been used for the transistor, although not incorrect, is certainly not in its simplest form. This paper will prove that the equivalent circuit obtained here will yield numerical solutions for its terminal voltage (as a function of an applied current) which are exactly equivalent to the solutions from the heat-flow equations for the temperature of the collector junction (as a function of an applied-power dissipation).

The validity of the equivalent circuit is based on the treatment of two physical quantities, heat capacity and thermal conductivity, according to the Cauer extension of Foster's reactance theorem, which states, in part, that any network of RC elements has a driving-point impedance that can be constructed of a single series string of parallel RC pairs.[3] The mathematical analysis presented in this paper leads to a thermal equivalent circuit with precisely this form. The mathematical analysis (given in the Appendix) of a very general model of a transistor will indicate that the number of parallel RC pairs in the series string should theoretically be infinite. The initial condition of the system, however, makes it a necessary condition that the series solution (where each term represents an RC pair) must be convergent. Therefore, it is shown that a series string with a finite number of parallel RC pairs can give a correct solution to any desired degree of accuracy. The determination of the precise number of RC pairs required for practical convergence will depend on an accurate K-factor measurement.

After the validity of the thermal equivalent circuit is shown, it will then be demonstrated how the circuit parameters can be determined experimentally. Then the important applications of the equivalent circuit for the circuit designer and the transistor designer will be discussed.

The thermal equivalent circuit

It is shown in the Appendix that for no applied-power dissipation at the collector junction of a transistor, the temperature decay at the junction will be given by

$$T_J(t) = \sum_{n=1}^{\infty} A_n \exp(-t/\tau_n), \qquad (1)$$

where the constants τ_n and A_n depend on the geometry, materials, and the initial conditions of the system. It will now be shown that a solution of this form may be obtained for the voltage decay at the terminals of a certain passive electrical network, and that the analogy presented in Table I may be applied to this electrical circuit to obtain an equivalent thermal circuit. We begin by asking what differential equation would give as a solution one of the terms of the series in Eq. (1), and find immediately that it would be

$$\frac{dT_n}{dt} + \frac{T_n}{\tau_n} = 0. \qquad (2)$$

If in Eq. (2) the constant τ_n is replaced by the product of the thermal resistance R_n and the thermal capacitance C_n, then Eq. (2) can be rewritten as

$$C_n \frac{dT_n}{dt} + \frac{T_n}{R_n} = 0. \qquad (3)$$

In the heat analogue, Eq. (2) can be considered the differential equation for the temperature drop across the terminals of a thermal resistor in parallel with a thermal capacitor, where the power dissipation applied to the terminals is zero. The solution of this equation is, of course,

$$T_n(t) = T_n(0)\exp(-t/R_nC_n), \qquad (4)$$

where $T_n(0)$ is the value of $T_n(t)$ at the time t equal to zero. If m such parallel RC networks were placed in series, as in Fig. 1, then the total temperature drop along the string would be the sum of the temperature drops across each of the thermal RC networks. Thus

$$T_J(t) = \sum_{n=1}^{m} T_n(t) = \sum_{n=1}^{m} T_n(0)\exp(-t/R_nC_n). \qquad (5)$$

The fact that the infinite-series solution, Eq. (1), must be convergent, as pointed out in the Appendix, is justification for approximating this solution by a finite series. To demonstrate this point, let us suppose that a source of constant power dissipation P_0 were applied to the terminals of the thermal equivalent circuit as in Fig. 1. After a sufficient time interval, the temperature drop across the terminals will be constant and would be given by

$$T_J = P_0 \sum_{n=1}^{m} R_n. \qquad (6)$$

Since T_J will be a constant with some finite value, then $\sum_{n=1}^{m} R_n$ must also be finite, and accordingly as m increases, the value of R_m must approach zero. In a practical sense it will always be possible to find a value for m such that the value of $P_0 \sum_{n=m+1}^{\infty} R_n$ will be essentially zero to any desired degree of approximation. It will be noted that the quantity $\sum_{n=1}^{\infty} R_n$ is identical to the K-factor of the transistor.

In general, the two accessible terminals in the thermal equivalent circuit of Fig. 1 represent the collector junction and the constant ambient temperature of the transistor. Unlike the more general equations for heat flow, such as Eq. (13), this thermal-equivalent-circuit approach can determine the temperature as a function of time at only one point in the transistor — the collector junction; and the best that could be anticipated is that an equivalent circuit could be found which would give the temperature at any number of discrete points in the transistor. The reason for this, of course, is that in electrical circuits one desires a solution for the voltage at only those points which are between the circuit elements, and the mathematics then leads to a total differential equation. On the other hand a solution for some quantity such as the temperature over a continuum of points must satisfy a partial-differential equation, such as the heat-conduction equation. But since the temperature at the collector junction is of primary interest, and the problem has been simplified by the invention of a thermal equivalent circuit from one which involved a boundary-value problem of the second order to a first-order total differential equation, then this limitation of the equivalent circuit is not a serious one.

We have shown that the solution of the heat-conduction equation for the temperature variation at the collector junction is identical to the solution obtained from the thermal equivalent circuit for the special case of no applied-power dissipation. It is then necessary to show that these solutions will be identical if a power dissipation $P(t)$ is applied to the collector junction. For this case the boundary condition Eq. (14a) of the problem would then be nonhomogeneous and given as

$$\frac{\partial T_1}{\partial n}\bigg|_{r_0} = p(t)f(r)\bigg|_{r_0}, \tag{7}$$

where the functions $p(t)$ and $f(r)$ would be such that

$$P(t) = \int_{r_0} p(t)f(r)\,dr, \tag{8}$$

where r_0 represents the collector-junction surface (see Appendix).

The boundary-value problem then becomes one of a homogeneous differential equation with a nonhomogeneous boundary condition. This problem is identical to having a nonhomogeneous differential equation with a homogeneous boundary condition as shown by a theorem found in many textbooks.[4] It is clear, then, that the solution obtained from the equivalent circuit when the driving function $P(t)$ is included at the terminals is identical to the general solution of the nonhomogeneous differential boundary-value problem.

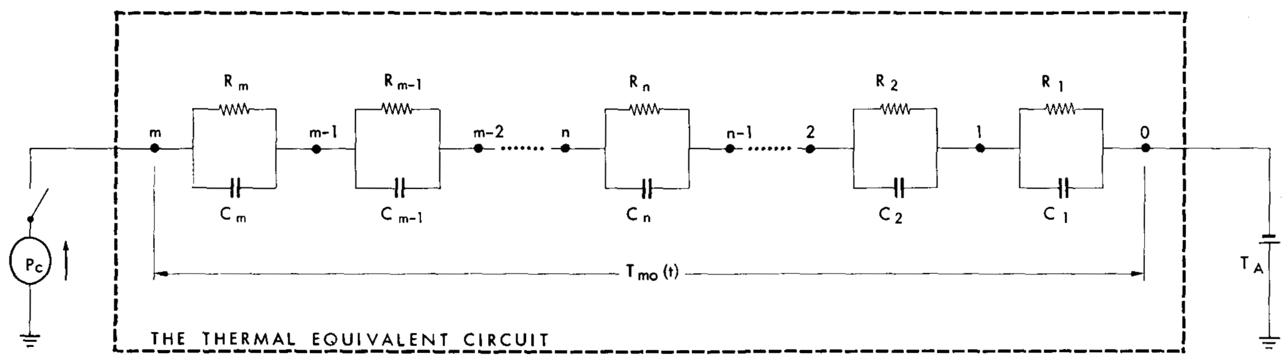
The Joule heating in the base region of the transistor has been neglected in this discussion since its magnitude is usually quite small compared to the power dissipation at the junction. When Joule heat cannot be neglected, it can be added to the differential equation as another driving function without affecting the validity of the thermal equivalent circuit.

Experiment to determine circuit parameters

The circuit parameters were determined experimentally from measurements of temperature decay of the collector junction. A constant power dissipation P_0 was applied to the collector junction of a transistor operating in a grounded-base configuration for a period of time sufficient to insure thermal equilibrium. At a time designated as zero the power dissipation was completely removed and the temperature decay of the junction to the ambient temperature recorded. This cooling curve of the junction was of the form predicted by Eq. (1), the analytical solution for the corresponding set of initial and boundary conditions. This agreement has been borne out by experimental data for six samples, and the details and results of one representative experiment will be reported.

The test unit was a low-power PNP alloy-junction transistor, IBM Type 13. The first step of the procedure was to calibrate some electrical characteristic of the junction as a function of temperature. The particular choice made here was the saturation current, which is fairly

Figure 1 **The thermal equivalent circuit for the collector junction of a transistor, along with the analogous circuit elements necessary to produce the cooling curve. $T_{mo}(t)$ is the temperature difference between collector junction and ambient as a time function. T_A is a battery representing constant ambient temperature. P_c is constant applied power for $t<0$.**



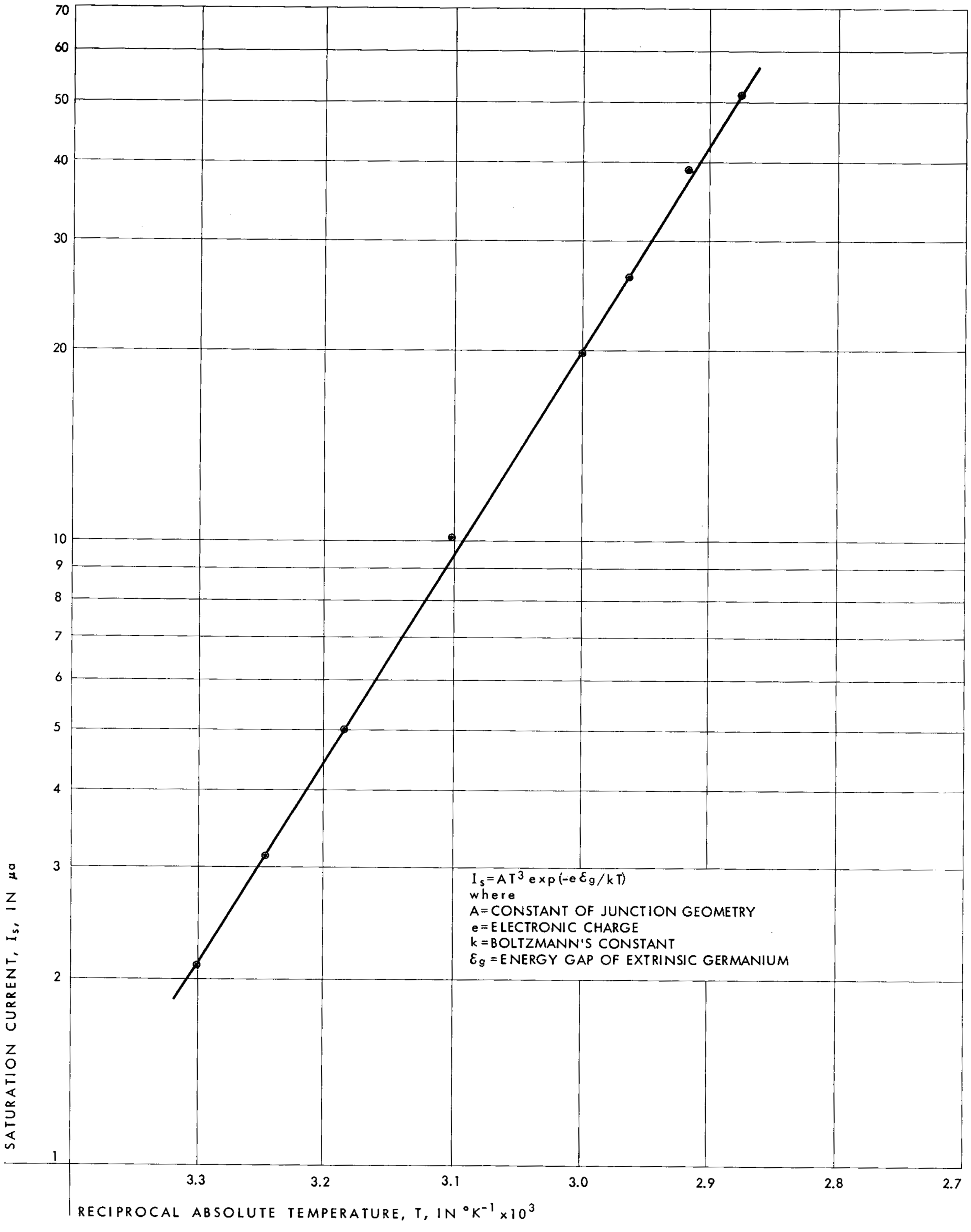


Figure 2 **Experimentally obtained curve for the saturation current of the test unit as a function of the reciprocal absolute temperature with the experimental points indicated. (The use of the straight line to smooth the data neglects the slowly varying T^3 factor.)**

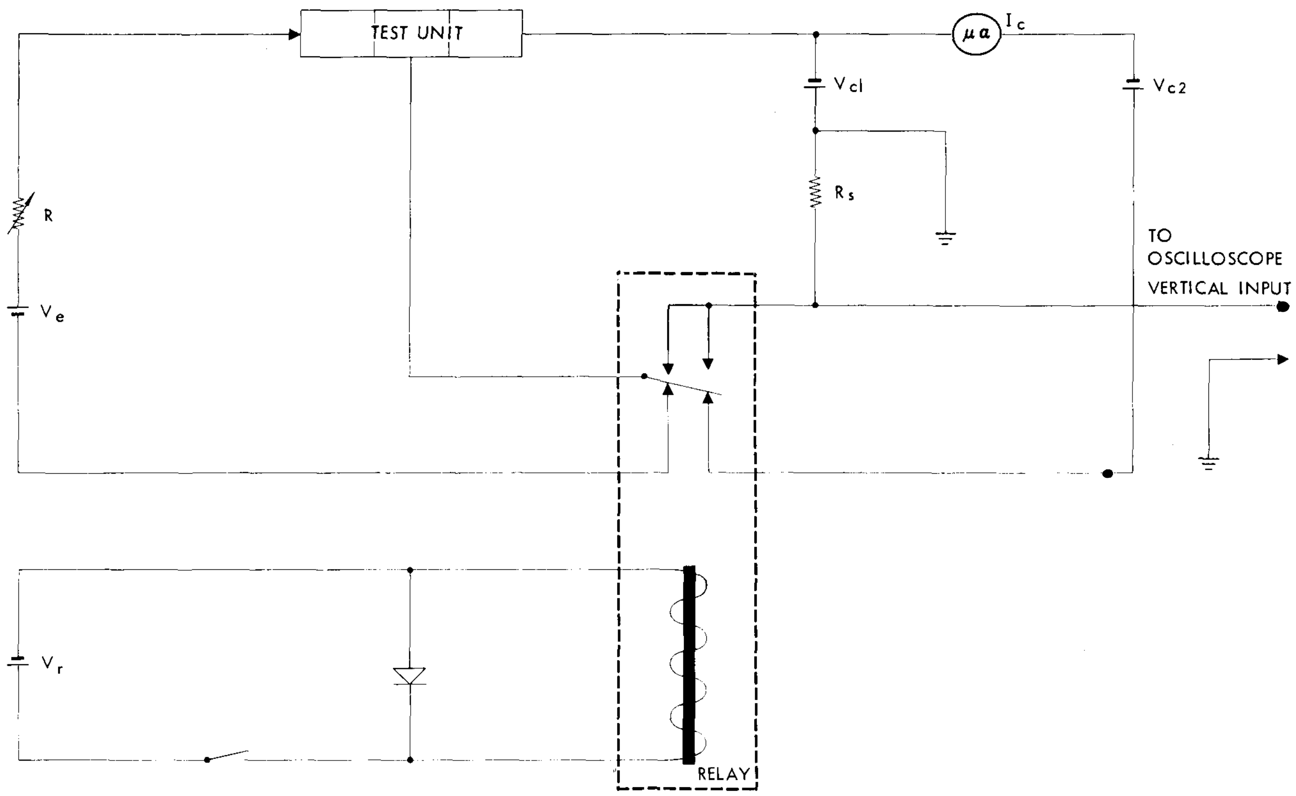


Figure 3 **The test circuit used in the determination of the cooling curve.**

independent of the reverse bias needed on the junction, provided this bias is between 0.2 and 2 volts.[5] Thus with the unit immersed in a temperature bath the saturation current I_s was measured at different temperatures and plotted in Fig. 2. The theoretical function for the temperature-dependent saturation current has been shown by DeWitt and Rossoff.[6] The value of energy gap $\mathcal{E}_g$ deduced from the slope of the curve in Fig. 2 is 0.65 ev.*

Next the test unit was placed in an environment of circulating silicone oil held at room temperature. The test unit was then connected to the circuit of Fig. 3. With the manual switch in its normally closed position, the collector junction dissipated power $P_0 = V_{c2}I_c$, controlled by the potentiometer R. Depressing the switch** opened the emitter circuit and applied the one-volt bias V_{c1} in series through the sense resistor R_s to the collector junction. The image of the vertical motion of the oscilloscope trace was recorded on a film moving horizontally past the face of the oscilloscope. With the vertical and horizontal distances on the film calibrated to indicate saturation current and time, respectively, this experimental curve was then converted into a temperature/time cooling curve by means of the calibration plot of Fig. 2. Subtracting from these data the constant ambient temperature T_A gave the curve T_{m0} in Fig. (4a). Since this plot approaches a straight line on semilog paper, it was assumed to be the contribution of the exponential term with the largest time constant. The difference between the cooling curve and the extrapolated straight line which it approached was then taken to be the contribution of the sum of the remaining exponential terms. This difference was plotted in Fig. (4b) as $T_m - T_{10}$. Again the plot approached a straight line, for which the exponential term was assumed to be the second in the series solution. This procedure of graphical analysis was continued until a curve was obtained that was considered to be an experimental straight line down to time zero. Thus the finite-series solution (which would approximate the infinite-series solution indicated by the heat-conduction equation) would contain four exponential terms for the test transistor in this particular environment. The value of the time constants and the initial values of each term have been taken from the plots, and the required calculations for circuit parameters of the thermal equivalent have been indicated on each plot. The equivalent circuit and the numerical conditions and results of this experiment are tabulated in Table II.

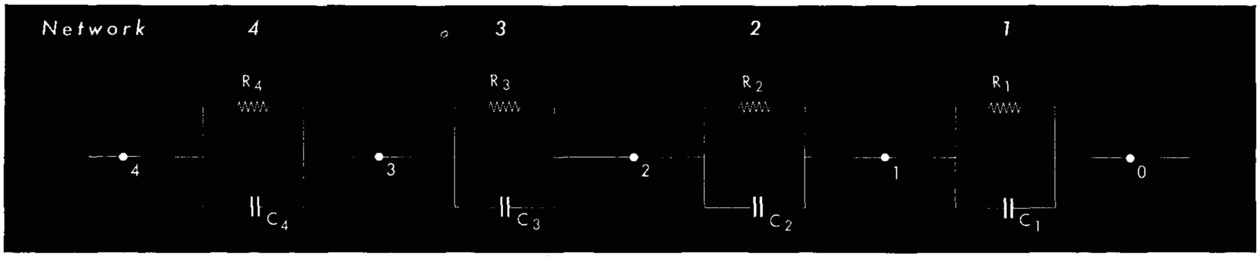
In this experimentally obtained equivalent circuit, the question arises as to whether or not there might be another network with a very small time constant which should also be included. Additional shorter time constants no doubt exist, but the real question is whether the corresponding RC networks — in the context of the analogy — can support any appreciable temperature

*This value compares favorably with 0.75 ev for Ge at 0°K, *American Institute of Physics Handbook,* McGraw-Hill Book Company, Inc., New York, 1957. P. 5-158.

**When the switch is depressed, the relay (Clare Mercury Wetted Contact) is transferred to its "off" position.

Table II **The thermal equivalent circuit of the test unit and the values of the circuit parameters.**

Test unit: PNP germanium alloy junction (IBM Type 13) transistor
Initial power dissipation: 150 mw
*Ambient temperature: 30°*C
Environment: Circulating silicone oil



Network n	$T_{(0)}$ °C	τ_n, sec	R_n, °C/mw	C_n, mw-sec/°C
1	2.83	6.05	0.0189	320
2	11.0	1.21	0.0734	16.5
3	2.90	0.300	0.0193	15.6
4	7.10	0.0117	0.0473	0.248

$K\text{-factor}=\sum_{n=1}^{4} R_n = 0.1589$

drop. To determine this, the total thermal resistance, or K-factor of the test unit, was measured at twelve different levels of constant power dissipation in circulating silicone oil at 50°C. The averaged value of these measurements indicated a total thermal resistance of 0.162°C/mw.

As indicated in Table II, the sum of the thermal resistances of the four networks obtained in this experiment is 0.159°C/mw. Thus the total of the omitted networks can have only an additional 0.003°C/mw thermal resistance. For a constant power dissipation of 150 mw, this represents a temperature drop of only 0.45°C. Since the transistor is rated at only a fraction of this power dissipation, the error in the junction temperature introduced by the omission of these networks will be very small. The above analysis depends, of course, on the ability to obtain a valid and independent measurement of the K-factor of the test unit, and for this the reader is referred to recent work by Reich[7] and by Nelson and Iwersen.[8]

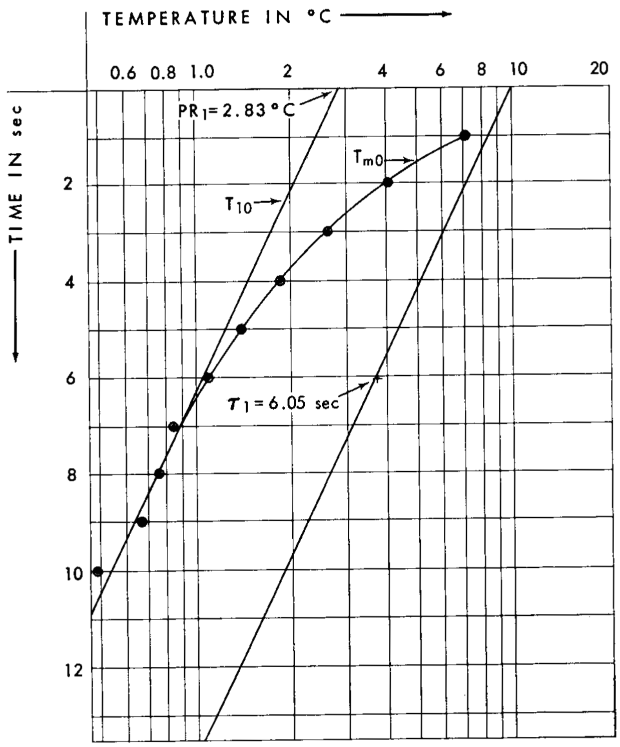
The power dissipation for the above K-factor measurement was not actually held constant but consisted of periodic rectangular pulses with an amplitude P_0, a repetition rate of 20 cps, and a duty cycle of 0.98. The waveform of the saturation current was observed during the 1-msec interval in which the power dissipation was "off." The appearance of the waveform during the first 20 μsec was attributed to circuit transients resulting from the switching, but the remainder of the waveform, which was almost linear and had only a very slight slope, could quite easily be extrapolated to the beginning of the time interval, thus indicating the temperature at the moment the power P_0 was interrupted.

It should also be pointed out that there is no theoretical objection to an experimental environment of air (either still or moving at constant velocity) at constant temperature, since at the external surface of the transistor a boundary condition based on Newton's law of cooling and its associated "heat transfer coefficient" includes the possibility of convection currents in any fluid medium. The environment of circulating silicone oil was used in this case because it was experimentally simple to create and measure.

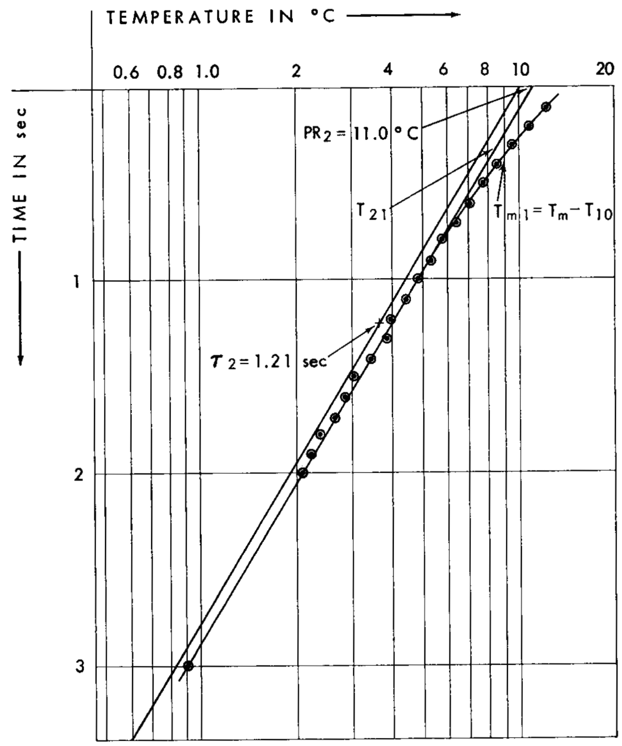
Application of the thermal equivalent circuit

Once the thermal equivalent circuit for a particular transistor or transistor type has been established by experiment, the circuit designer may simply determine the temperature response of the collector junction as a function of the applied-power dissipation. This is accomplished by imagining the known function of power dissipation to be a current function applied to the terminals of the equivalent circuit, and by solving for the voltage response across the terminals. If, for instance, the applied-power dissipation were sinusoidal, then the collector temperature would be expected to vary sinusoidally with some predictable amplitude and phase shift with respect to the applied power. The temperature response to periodic power pulsing and to single pulses can easily be established. In fact the temperature response of the collector junction for any known, applied-power

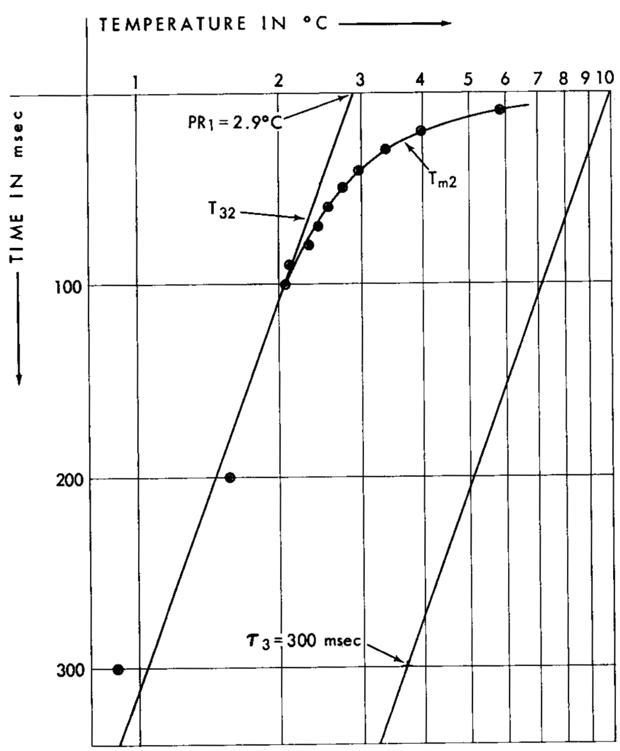
Figure 4 **Graphical analysis of the cooling curve for the test unit.**



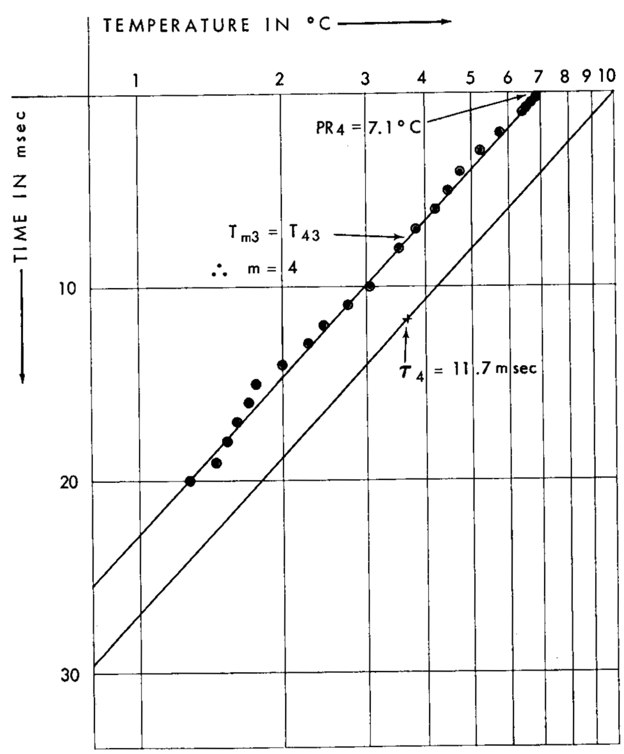
4a *Network 1*
$R_1 = 0.0189$°C/mw $C_1 = 3.20 \times 10^2$ mw-sec/°C



4b *Network 2*
$R_2 = 0.0734$°C/mw $C_2 = 16.5$ mw-sec/°C



4c *Network 3*
$R_3 = 0.0193$°C/mw $C_3 = 15.6$ mw-sec/°C



4d *Network 4*
$R_4 = 0.0473$°C/mw $C_4 = 0.248$ mw-sec/°C

dissipation can be determined by the analysis of a simple series of RC networks.

As an example, consider the case of switching circuits where the transistor might be subjected to periodic power pulses, i.e., for a given interval of time a (Fig. 5a) a constant amount of power P_0 is dissipated by the transistor and then the device is turned off for the remainder of the period p. Let b denote the power-off time. With the aid of the thermal equivalent circuit and the rules for circuit analysis it is now possible to determine the temperature response of the junction for this particular function of applied-power dissipation. To simplify this example, only the steady-state response will be obtained; however, the transient response for the first few seconds of pulsing could be obtained by a more complete circuit analysis. It is observed from Fig. 1 that temperature variations across only a single RC network need be analyzed since the others will be identical in form — differing only in their particular values of R and C. When the calculation for this one network is complete, it can then simply be added to the response of the others in order to get the total temperature response. The steady-state temperature response of the n^{th} thermal RC network is given by the following two functions:

$$T_n(t)=[P_0R_n-T_n(\text{min})][1-\exp(-t/R_nC_n)]+T_n(\text{min}) \tag{9a}$$

for the time interval when $P(t)=P_0$, and

$$T_n(t)=T_n(\text{max})\exp(-t/R_nC_n) \tag{9b}$$

for the time interval when $P(t)=0$, where $T_n(\text{min})$ and $T_n(\text{max})$ are constants representing the minimum and maximum temperatures, respectively, across the network during a cycle. Since the value of $T_n(\text{min})$ is obtained at the end of the power-off time, its value in terms of $T_n(\text{max})$ may be obtained by putting the value b into Eq. (9b) for t, obtaining

$$T_n(\text{min})=T_n(\text{max})\exp(-b/R_nC_n) \tag{10}$$

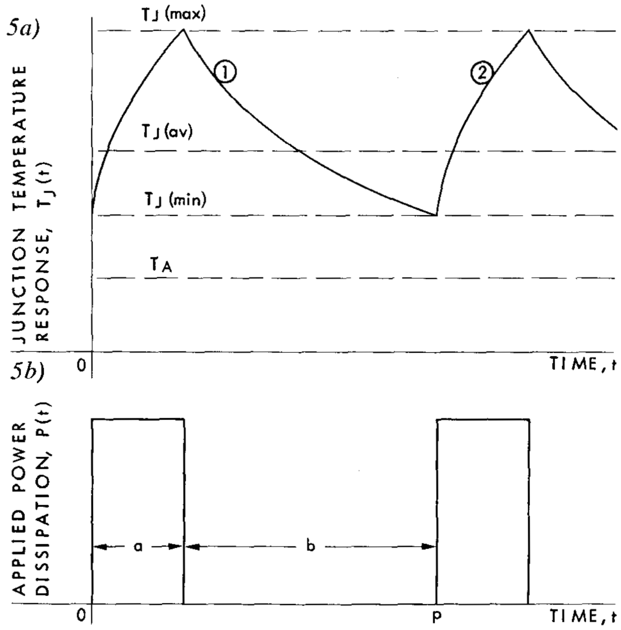
The value of $T_n(\text{max})$ can be obtained from Eq. (9a) after the time interval a. Thus, after substituting Eq. (10) into Eq. (9a) and then replacing the dependent variable $T_n(t)$ with $T_n(\text{max})$ and the independent variable t with a, the resulting expression may then be solved for $T_n(\text{max})$, yielding

$$T_n(\text{max})=P_0R_n\frac{1-\exp(-a/R_nC_n)}{1-\exp(-p/R_nC_n)}\,. \tag{11}$$

The constants of Eqs. (9), $T_n(\text{min})$ and $T_n(\text{max})$, can now be obtained from Eqs. (10) and (11), and by summing Eqs. (9) over the m networks the instantaneous junction-temperature response is determined. Figure 5b illustrates this steady-state response of the junction tem-

Figure 5 **The steady-state temperature response of the collector junction for an applied-power dissipation of rectangular periodic pulses.**

$$T_J(\text{av}) =\sum_{n=1}^{m} T_n(\text{av})=\frac{a}{p}P_o\sum_{n=1}^{m}R_n-\frac{P_o}{p}\sum_{n=1}^{m}R_n{}^2C_nT_n(\text{max})[1-\exp(-a/R_nC_n)][1-\exp(-b/R_nC_n)]$$



$$T_J(\text{max}) =\sum_{n=1}^{m} T_n(\text{max})+T_A=P_o\sum_{n=1}^{m}R_n\frac{1-\exp(-a/R_nC_n)}{1-\exp(-p/R_nC_n)}+T$$

$$T_J(\text{min}) =\sum_{n=1}^{m} T_n(\text{min})=\sum T_n(\text{max})\ \exp(-b/R_nC_n)$$

$$① \; T_J(t) =\sum_{n=1}^{m} T_n(\text{max})\exp(-t/R_nC_n)$$

$$② \; T_J(t) =\sum_{n=1}^{m}\{(P_oR_n)-T_n(\text{min})\ [1-\exp(-t/R_nC_n)]+T_n(\text{mi}$$

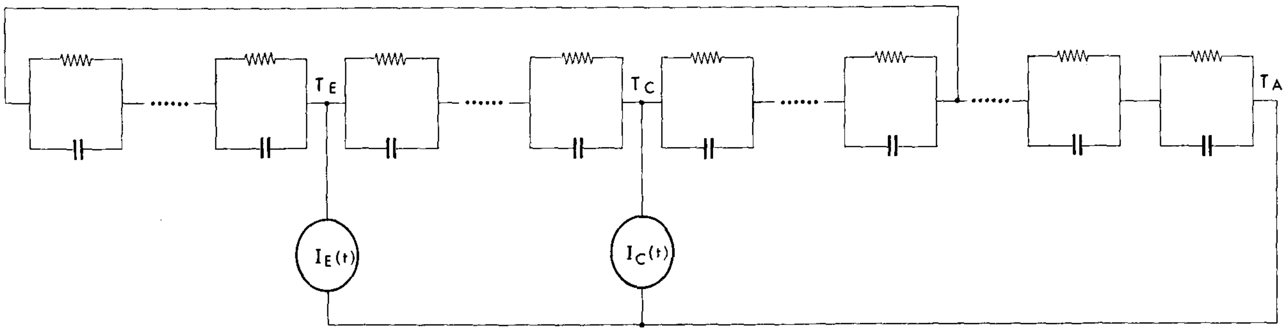


Figure 6. **A proposed extension of the thermal equivalent circuit for a transistor to include the emitter junction.**

perature for an excitation of periodic power pulses. Also shown on this plot is the solution for the time average of the junction temperature, and it will be noted that this average is always less than the average obtained by multiplying the average power dissipation $(a/p)P_0$ by the K-factor

$$\left(\sum_{n=1}^{m} R_n\right).$$

The thermal equivalent circuit might also be extended to include the temperature response of the emitter junction as suggested by the circuit configuration of Fig. 6. The circuit parameters of this network might be analyzed by determining the cooling curve of the emitter and collector junctions separately and, from the difference of these, establishing the cooling curve of the emitter with respect to the collector junction. Since the question of the emitter temperature caused by power dissipation of the collector frequently arises in transistor work, this equivalent circuit would be worthwhile. If the power dissipation at the emitter can be neglected, then one need only measure the thermal resistance between emitter and collector junction, since by Eq. (16) all points have the same time constants.

For the transistor designer who has attempted an analysis of the heat-conduction equation for a particular geometry, the experimentally obtained time constants of the thermal equivalent circuit can be used as a check on the results of the analysis. Since in general the completion of the problem in the Appendix for numerical results may be very difficult, it is extremely helpful in such an analysis to be able to make valid simplifying assumptions. When the boundary conditions of the problem are imposed on the general solution Eq. (16), a characteristic equation will evolve which determines the permissible values, or eigenvalues, of the separation constant λ_1^2. Obtaining this equation and subsequently solving it for the eigenvalues is straightforward in principle only. For example, to find this equation for the test unit reported on earlier it would be necessary to know the geometrical dimensions in detail, since it is necessary to solve for the space part of the solution $R(r)$. But if it can be assumed that certain regions of the transistor are approximately independent of the remainder of the device, it might be possible to break the complex problem into several simpler ones. To obtain an intuitive understanding, consider the linear flow of heat through several materials in succession, where the effective time constants for each region differ from the time constants of the others by several orders of magnitude. Then when a thermal gradient is established through these materials in series and the heat source is removed, one would expect the temperature drop across the material with the shortest time constants to become effectively zero before the others have started their temperature decay. During the remainder of the temperature decay of the system the material with the short time constants will simply assume a constant temperature throughout which is equal to that at its boundaries.

This approximate method will now be applied, in a crude manner, to obtain analytically the time constants of the test unit. The transistor is assumed to have right-circular geometry with the active elements occupying a very small volume at the center, the bulk of the thermal system (the desiccant powder and the case) to be the second region, and the third region to be the stationary film of oil adjacent to the case. Since the case is not actually circular, its radius is approximated by the dimension 0.3 cm. For geometries involving right-circular symmetry it can be shown that the space function $R(r)$ turns out to be a Bessel-type function and that the eigenvalues are determined by the characteristic equation $J_0(r\lambda_i)=0$. The values of the argument of this Bessel function which give it a zero value are tabulated, and when r is divided into these tabulated values the eigenvalues λ_i are obtained. The time constants then are simply $1/\alpha\lambda_i^2$, and by assuming a diffusivity α of 0.01 cm^2/sec a series of time constants can be computed, the first three of these being 1.5 sec, 300 μsec, and 65 μsec. The first two agree very well with the experimental values and it may be considered that the series solution for this region converges in only two terms. The smallest time constant in Table II must be contributed by the comparatively small central region and the series for this region apparently converges in only one term. Since the details of internal structure are not very well known in this case, an analysis of this region cannot be made here. The value in making such an analysis is that if a valid experimental K-factor measurement cannot be performed, the experimental time constants will serve to check and improve the analytical solution. It is then possible to determine as many short time constants and coefficients as necessary and then to decide how many terms must be included in the series for practical convergence.

The six-second time constant in Table II can be associated with the third region, that of the environment. The time constants for this series (convergent in one term for this test unit) will depend strongly on the value of the heat-transfer coefficient discussed earlier. To compute the value of this constant for a particular boundary or interface is impossible in most instances. But an improvement of the coefficient as a result of redesign may be observed readily in a smaller value of the constant R_1 in Table II. This same argument can be applied to the other regions and the results can ultimately be used to obtain optimum power-dissipation ratings in package designs.

One further application of the thermal equivalent circuit might be to check the uniformity of manufactured transistors by determining the variations in the thermal circuit parameters from unit to unit. Defects such as poor thermal bonds between germanium wafers and base tabs might be rapidly detected in this way.

Conclusions

By considering the academic approach to the problem of heat conduction in a transistor, a thermal equivalent circuit for the transistor was derived. The validity of this equivalent circuit was established mathematically, and an experiment to determine the circuit parameters was described. The temperature response of a semiconductor junction to an applied-power dissipation can then be determined by analyzing this circuit rather than by the more difficult solution for the boundary-value problem associated with the partial-differential equation of heat flow. Thus a practical approach has been developed for the problem of determining the instantaneous value of the junction temperature of an operating transistor as a function of time. The usefulness of the thermal equivalent circuit has also been indicated for the development of optimum power-dissipation ratings in transistor designs.

The graphical analysis of the cooling curve of a junction presented here is not necessarily the best method for determining the circuit parameters of the equivalent circuit. If this method should be used for a particular transistor which has significant short-time-constant networks, care must be taken in the construction of the test circuit so that the cooling curve is not masked by electrical transients for very small time intervals. Also some reliable method of determining the K-factor of the transistor must be used to determine the completeness of the equivalent circuit obtained.

Appendix

The purpose of this section is to demonstrate that the temperature decay at the collector junction of a transistor for the special case of no applied-power dissipation can always be expressed in the form

$$T_J(t) = \sum_{n=1}^{\infty} A_n \exp(-t/\tau_n) \qquad (12)$$

where in principle, at least, the constants τ_n and A_n may be determined from a knowledge of the geometry, constitution, and initial state of the system, which includes the environment. In Eq. (12), $T_J(t)$ must be regarded as some sort of a space average or mean temperature taken over the region of the junction.

Consider a very general model of a transistor system consisting of three regions labeled 1, 2, and 3. Region 1 is a homogeneous solid material with a heat conductivity of k_1, a thermal diffusivity of α_1, and continued in this region is a surface which may act as a source of heat. Region 1 is completely surrounded by Region 2, which may consist of many materials (solid or liquid) in any arrangement but does not contain any heat sources. Region 3 will completely surround Region 2 and the only requirement for this region will be that it is all at essentially a common temperature which we will arbitrarily call zero. In this model we will let r be a three-dimensional coordinate denoting a position in the space of the system (or model). In particular, we will let r_0, r_{12}, and r_{23} represent those values of r which form the surfaces of the heat source, of the interface between Regions 1 and 2, and between Regions 2 and 3, respectively. It is intended in this model that the surface r_0 represent the collector junction, that Region 1 is the base region of a transistor, that Region 2 is all other transistor materials, and that Region 3 is the environment.

Since Region 1 is a solid material we may write, when r is in this region, that

$$\frac{\partial T_1(r,t)}{\partial t} = \alpha_1 \nabla^2 T_1(r,t), \qquad (13)$$

subject to the boundary conditions

$$\left.\frac{\partial T_1}{\partial n}\right|_{r_0} = 0 \qquad (14a)$$

$$\left. T_1 \right|_{r_{12}} = \left. T_2 \right|_{r_{12}} \qquad (14b)$$

$$\left. k_1 \frac{\partial T_1}{\partial n}\right|_{r_{12}} = \left. P_2 \right|_{r_{12}}, \qquad (14c)$$

where $T_1(r,t)$ is the temperature at the point r (in Region 1) and at the time t. The notation for the boundary conditions (Eqs. 14) implies that (a) no heat flows across the surface r_0, (b) the temperature at the interface between Regions 1 and 2 is continuous, and (c) that the rate at which heat is leaving Region 1 is the same as the rate at which heat is entering Region 2. The particular solution which we are seeking from Eqs. (13) and (14) is $T_1(r_0, t)$, because from this $T_J(t)$ may be obtained.

By the method of separation of variables it may be assumed that the general solution of Eq. (13) can be found by letting

$$T_1(r,t) = \theta_1(t) R_1(r), \qquad (15)$$

where the functions θ_1 and R_1 must yet be determined.

By substituting Eq. (15) into Eq. (13) we are led, in the usual manner, to the general solution for the temperature in Region 1, which is

$$T_1(r, t) = \exp(-t/\alpha_1\lambda_1^2)R_1(r), \qquad (16)$$

where $-\lambda_1^2$ is the separation constant (the positive and zero values of this constant having been systematically eliminated), and the function $R_1(r)$ is determined by solving the differential equation

$$\nabla^2 R_1(r) + \lambda_1^2 R(r) = 0 \qquad (17)$$

subject to the boundary conditions of Eq. (14). It turns out, of course, that the boundary conditions do not determine a unique value for the constant λ_1^2 in Eq. (17), but instead allow this constant to assume an infinite number of eigenvalues, each of which can lead to a different solution for $R_1(r)$. Thus the particular eigenvalue $(\lambda_1^2)_n$ will give as a solution the eigenfunction $R_{1n}(r)$. A complete solution must be the sum of the individual solutions; and, in particular, the solution at the surface r_0 will be

$$T_1(r_0, t) = \sum_{n=1}^{\infty} \exp[-\alpha_1 t/(\lambda_1^2)_n] R_{1n}(r_0). \qquad (18)$$

The space average of $T_1(r_0, t)$ over the surface r_0 will be obtained by determining the space average of $R_{1n}(r_0)$. Then, by letting $\tau_n = \alpha_1(\lambda^2{}_1)_n$ and $A_n = \overline{R_{1n}(r_0)}$, Eq. (18) may be rewritten as

$$T_J(t) = \overline{T_1(r_0, t)} = \sum_{n=1}^{\infty} \overline{R_{1n}(r_0)} \exp[-\alpha_1 t/(\lambda_1^2)_n]$$

$$= \sum_{n=1}^{\infty} A_n \exp(-t/\tau_n) \qquad (19)$$

which is Eq. (12). It will be noted that in the derivation of Eq. (19) it was unnecessary to make reference to Regions 2 and 3. This simply means that the *form* of Eq. (19) is quite independent of these regions. It is not true, however, that Eq. (19) is unaffected by Regions 2 and 3. If one were to attempt, by analytic methods only, to establish the exact values of the constants A_n and τ_n, it would become immediately apparent that they are very strongly dependent on the geometry, constitution, and initial state of Regions 2 and 3. It should also be pointed out that since the initial condition for $T_J(t)$ indicates a non-infinite temperature, this is a necessary condition for the series of Eq. (19) to converge.

References

1. K. E. Mortenson, "Transistor Junction Temperature as a Function of Time," *Proceedings of the IRE,* **45,** No. 4, 504 (April 1957).
2. C. D. Simons, "The Thermal Response Characteristics of the SBT and the MAT," *Application Report #321,* Lansdale Tube Company, Lansdale, Pennsylvania (October 3, 1957).
3. The author is indebted to Mr. D. DeWitt for pointing out this fact. The theorem may be found in Guillemin, *Communication Networks,* Wiley 1935, Chap. 5, Sec. 9.
4. C. H. Page, *Physical Mathematics,* D. Van Nostrand Company, Inc., Princeton, New Jersey, Chap. 7, Sec. 1 (1955).
5. K. E. Loofbourrow and J. Ollendorf, "Equipment for Measuring the Junction of an Operating Transistor," *Transistors I,* RCA Laboratories, Princeton, New Jersey, 353 (1956).
6. D. DeWitt and A. L. Rossoff, *Transistor Electronics,* McGraw-Hill Book Company, Inc., New York, first edition, 1957. Chap. 2, p. 76.
7. Bernard Reich, "Measurement of Transistor Thermal Resistance," *Proceedings of the IRE,* **46,** No. 6, 1204 (June 1958).
8. J. T. Nelson and J. E. Iwersen, "Measurement of Internal Temperature Rise of Transistors," *Proceedings of the IRE,* **46,** No. 6, 1207 (June 1958).

Revised manuscript received October 24, 1958