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An Analysis of the Operation of a Persistent-Supercurrent Memory Cell

Abstract: From a thin-film persistent-supercurrent memory cell is abstracted a particular theoretical model which is discussed exactly and in detail. The behavior predicted is in good agreement with that of experimental devices built to be as closely like the model as was possible at the time.* Partly because of the simplicity of the analysis and the rather complete understanding thus gained, this model may prove to be useful for the design of large-scale memory and computing systems. For a single cell, a memory cycle time of 5 millimicroseconds should be achievable, and for a large memory perhaps 10 millimicroseconds.

Introduction

The pioneering experiments of J. W. Crowe on a particular form of persistent-supercurrent memory, reported in the article on page 294 of this issue, open a new field of investigation and allow construction of devices not limited by previous considerations of size, speed, expense, and method of assembly. In order to make rapid progress we must, I believe, take the same step which occurred in the transistor field, in which the junction transistor has largely supplanted the point-contact device because it is better understood, and has therefore been the subject of more invention and experimental confirmation. Thus there is in technology, as well as in physics itself, ample precedent for the abstraction of an idealized model from an imperfectly understood device or phenomenon. This model, to be useful, should be simple enough to be analyzed exactly, should show the phenomenon to be discussed, should be consistent in that it violates no physical laws and therefore can actually be constructed and tested. The common alternative mode of discussion of complex phenomena is to make approximate descriptions and solutions of the complicated case. The accuracy and propriety of these approximations is usually in doubt; and it appears to the author to be preferable to describe a model which, although not exactly the device or phenomenon to be explained, is just as good a device, and can be discussed separately. Experiment can then show whether the conclusions drawn from the model are applicable to the original device. In many cases it is possible to construct the model, as in the case of the junction transistor, and it may then serve as a device in its own right, with the advantage that it is easy to understand and to calculate the effects of various changes and improvements. It is this method of approach which we adopt here.

The question of models is perhaps particularly difficult in the case of superconductivity because the fundamental theory itself does not yet exist. The mechanism of the transition from the superconducting to the normal state and back again is imperfectly understood. In addition to attacking these fundamental problems, we may make further progress on superconducting devices by using as components of these devices those elements which we believe to understand macroscopically; that is, the models may be imagined to be constructed of small wires, the behavior of which in transverse and longitudinal magnetic fields is fairly well understood. In truth our devices are made of thin films, sometimes thinner than a penetration depth. It will be seen, however, that the behavior predicted for the model constructed of fine wire is largely confirmed by that made of thin films.

The model

The original experiments of J. W. Crowe were done with coils or wires over continuous superconducting films. As the current in the drive wire was increased, the film became normal-conducting in spots and then returned to the intermediate state, having allowed some flux to penetrate. The unknown current distribution in the film, or its equivalent magnetic field, is truly a field, a function in space; the problem is accordingly very difficult and, in fact, not well defined. The adoption of

*This type of information storage was invented by J. W. Crowe, whose experimental work is reported on page 294 of this issue.

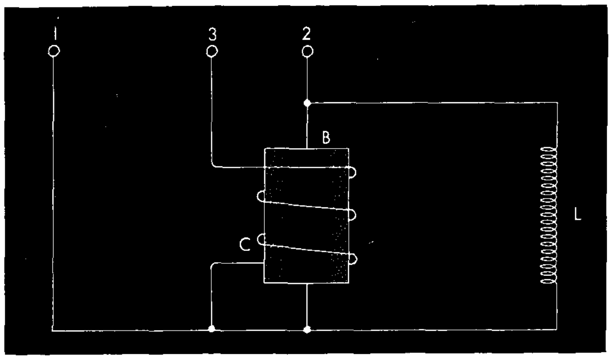

Figure 1 **Convenient circuit for setting up persistent supercurrents of smoothly controllable magnitude.**

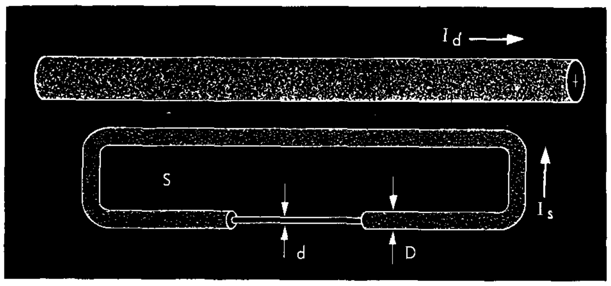

Figure 2 **Persistent-supercurrent storage loop.**

a thin superconducting film is convenient in fabricating a computer by evaporation of metals in vacuum and since the film is useful for shielding the sense wires of the memory from the drive wires, it is retained in the model. However, the film allows current to flow anywhere in its plane, so that there is therefore no unique state of current flow once the critical field has been surpassed anywhere. This condition is an extreme nuisance to be eliminated in the model. We therefore choose for our model a geometry in which the continuous film always remains superconducting and the current is therefore determined by the condition that the magnetic field component normal to the superconductor should be zero. This is an ordinary linear problem in magnetostatics and causes little difficulty. In a useful device, however, some portion must be driven into the normal state, so that after the application and removal of drive currents, a circulating current may persist in a superconducting loop to retain the information stored. To take a simple model, we assume that the only conductors which are switched from the superconducting to the normal state are very small wires, in which the current flow is entirely determined. In this manner one is led from the continuous film for storage of the persistent supercurrents to a film with a single rectangular hole with a very small wire or narrow ribbon across it, so that the magnetic field is entirely determined by the magnitude of the current in the wire bisecting the hole.

In persistent-supercurrent work, there are several possibilities for the control of the superconducting status of material; the external magnetic field applied to this particular conductor may be very large compared to the surface field produced by the currrent which is induced in this conductor by external sources; on the other hand the self-field due to the induced or the conducted current may be large compared to the external control field; or there may be some mixture of these two conditions.

An example of the first method of control is the practical device shown in Fig. 1. The loop comprising inductance L (which may be simply a straight wire) and control link B is maintained below the superconducting transition temperature of the materials involved. All joints are superconducting. Link B may be held in the normal state by current in coil C, which may be, but need not be, superconducting itself. Thus if B is held resistive by current in C while current I_s is applied to lead 2, this current will transfer with time constant L/R to flow in the zero-resistance inductance L. Should C now be de-energized and lead 2 disconnected from the supply at room temperature, current I_s will continue to circulate through L and B. In this manner a persistent supercurrent of any desired magnitude may be stored in L. This persistent current may be used as the excitation for an "air"- or iron-core magnet for information storage, for energy storage, etc. This exemplifies the principle of control by an external field.

In this circuit one is able to create, at temperatures below the superconducting transition point of a given material, a persistent supercurrent of any desired magnitude up to a certain maximum. Thus one has for low-temperature work an essentially permanent magnet of accurately and smoothly variable intensity, which requires no power to maintain the magnetic field, as is also the case with ordinary permanent magnets, and the magnetic flux of which is exceedingly constant, the thermal expansion of materials being extremely small and the temperature control very good in the liquid helium range. Dr. H. A. Reich and the author have in fact applied this persistent-supercurrent method to the control of the heat switches in the adiabatic demagnetization refrigerator.

In the present instance, however, one of the prime considerations is to avoid the use of coils or other methods of increasing the magnetic field locally, so that the whole device may be made by vacuum evaporation of thin films of superconductors. In this way the magnetic energy of the system is also held to a minimum, allowing very high speeds of operation. It is therefore more pertinent to consider a method of control in which the external magnetic field acting on the superconducting wire is completely negligible compared to the self-field due to the wire current which is induced by changes of

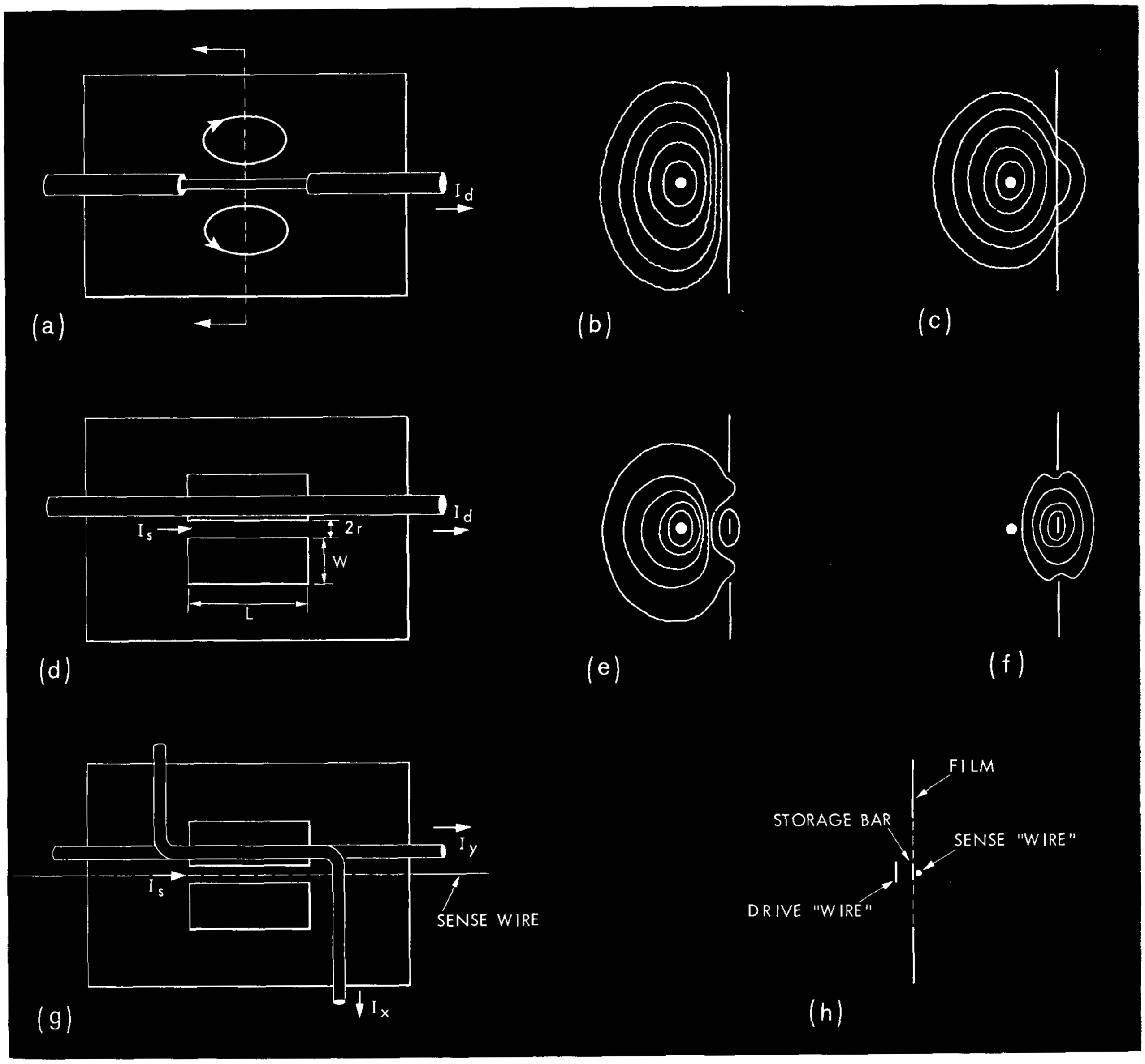

Figure 3 **Persistent-supercurrent storage cells using superconducting films.**

a) A wire with region of reduced diameter over a superconducting film sees its reflection in the film, which reduces the field normal to the film to zero. The actual screening currents in the film are sketched.

b) For small currents with drive wire, screening currents prevent field from penetrating the film.

c) Larger drive currents cause film to go normal, allowing field to penetrate, thereby reducing field and allowing the film to return to the "intermediate state." The distribution of current is a problem of an ill-defined field theory and is impossible to calculate.

d) Modification of (a), with fine wire stretched across hole in film. As in Fig. 2, the wire is so fine that it is the self-field of induced current that drives it into the normal state. There is now no problem of unknown current distribution since the film itself is always superconducting.

e) Barred-hole in (d) before break-through.

f) Barred-hole of (d) after break-through and removal of drive current, showing persistent supercurrent (trapped flux).

g) Barred-hole memory with coincident-current selection.

h) Detail of sense wire location to achieve almost unity coupling to eliminate half-select noise.

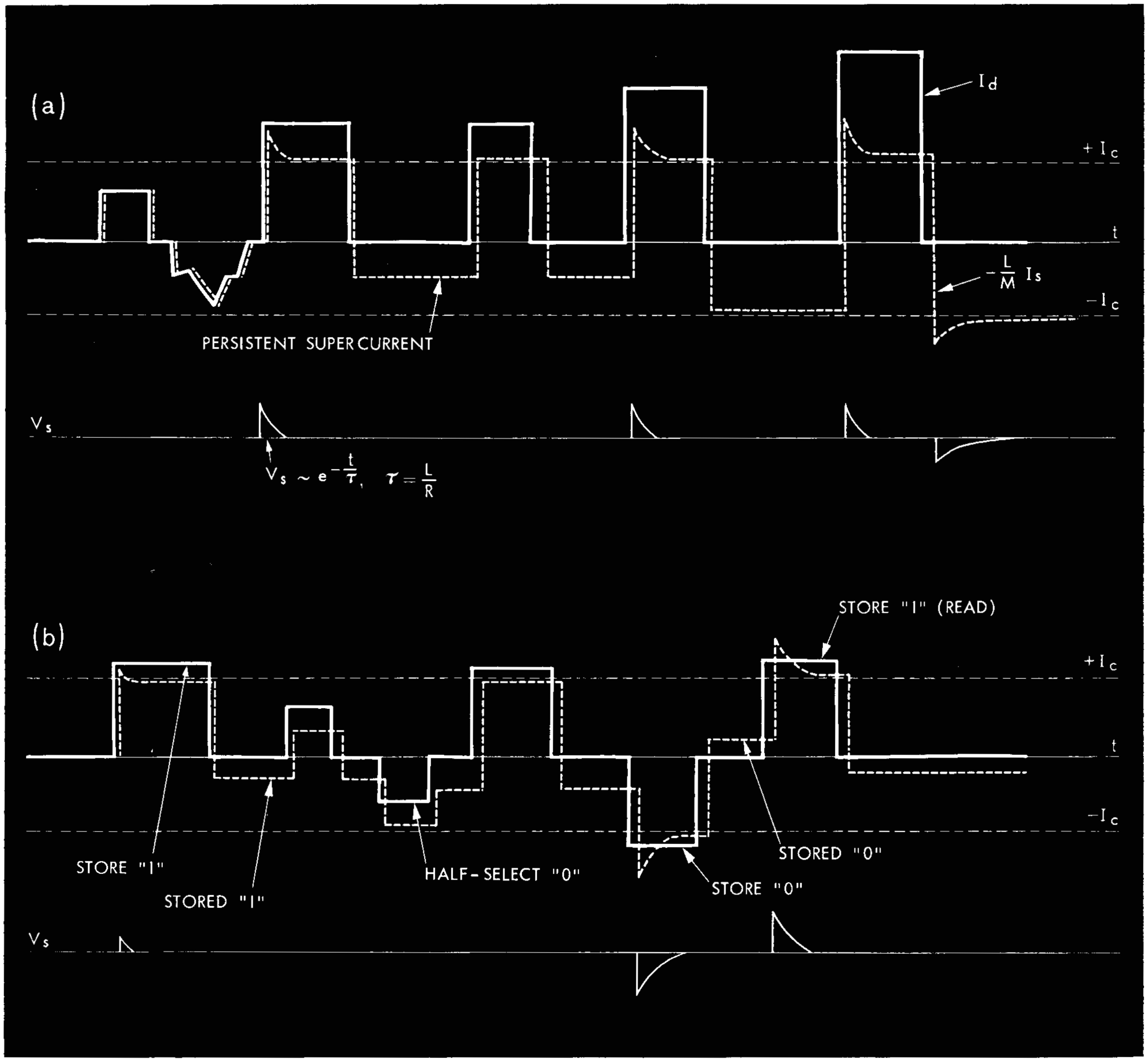

Figure 4 **Time variation of loop current for various drive pulses.**
In (a) we plot drive current I_d and $-(L/M)I_s$ (dotted). The output of the sense wire is shown below. I_S is shifted slightly toward right for clarity. In (b) is shown full drive current 1.25 critical, demonstrating that the half-selects do not disturb the stored information nor give sense outputs. One sees also that the rule: "to read, store '1'," yields a sense output for a stored "0" but none for a stored "1".

drive-wire current. It is easy to see that if in a small section of circuit coupled inductively to some drive wires the conductor diameter is sharply reduced, the induced current will not change; but the local magnetic field produced by the induced current will increase inversely as the diameter of the wire in this small region (Fig. 2). The drive wire carrying current I_d serves only to induce a current in the storage loop S. [In the superconducting state: $L(dI_S/dt)+M(dI_d/dt)-0=(d\phi_S/dt)$.] It is the *self-field* of the loop current which drives the loop normal at the constriction and is responsible for the storage of information and persistent currents. It is, of course, even possible to induce currents in S and to store persistent supercurrents in S with drive-wire configurations which have strictly zero external field, i.e., drive-wire wound on the surface of a torus which links S. This geometry is an example of control by self-field of an induced current.

By making the wire diameter small enough for a short region, one may make the self-field exceed the bulk critical field for the superconductor even in very small external magnetic fields. This method of control fits very well with the idealization of our model. In order to reduce the problem to that of a single variable as a function of time, we have localized the current in a very fine wire, the only portion of the circuit which is to be driven normal. The evolution of the model is shown in Fig. 3.

We shall first discuss the isothermal behavior of the storage cell shown in Figs. 2, 3d, 3g, 3h, which depict a persistent-supercurrent storage cell consisting of a superconducting loop S; i.e., loop S of critical current I_c (by virtue of the section of fine wire or narrow film included in the circuit) coupled by a mutual inductance M to a drive wire carrying current $I_d(t)$. Coupled to the storage loop with unity coupling factor is a sense wire, in which the induced voltage V_s will serve to indicate ("read") information residing in a storage cell. If the self-inductance of the loop is L, and the resistance present in the loop for loop currents $I_S>I_c$ is R, then the system obeys the following simple differential equations:*

For $I_S<I_c$:

$$M\frac{dI_d}{dt}+L\frac{dI_S}{dt}=0=\frac{d\phi_S}{dt} \tag{1}$$

$$V_s=M\frac{dI_d}{dt}+L\frac{dI_S}{dt}=\frac{d\phi_S}{dt}=0 \tag{2}$$

For $I_S>I_c$:

$$M\frac{dI_d}{dt}+L\frac{dI_S}{dt}+I_SR=0 \tag{3}$$

$$V_s=M\frac{dI_d}{dt}+L\frac{dI_S}{dt}=\frac{d\phi_S}{dt}=-I_SR \tag{4}$$

Typical solutions of these equations (for drive-current waveforms suitable for use in a memory) are shown in Fig. 4.

Thus, a drive current I_cL/M is required to store any persistent current at all, and a drive current $2I_cL/M$ will store a persistent supercurrent of magnitude I_c. In a planar memory with the drive-wire composed of two wires or ribbons (x- or y-select wires as in Fig. 3g), each carrying current $I_d/2$ when pulsed, the requirement is that the simultaneous presence of two currents of magnitude $I_d/2$ will store information, but the presence of only one such current will not disturb the existing information nor induce a signal in the signal wire. (In the present instance a stored "1" is a positive loop current of magnitude I_s, while a "0" is a negative loop current of the same magnitude.) The first condition requires that

$$I_d>\frac{L}{M}I_c \qquad \text{while the second requires} \tag{5}$$

$$\frac{I_d}{2}<\frac{2L}{M}(I_c-I_d) \qquad \text{or, together,} \tag{6}$$

$$I_c<\frac{M}{L}I_d<\frac{4I_c}{3}. \tag{7}$$

For drive currents within this 33% range we have thus a close analog to a coincident-current core storage. It is of interest to compute the switching time for such a memory bit: In Fig. 3d let $w=0.1$ cm, $l=0.1$ cm, $r=0.01$ cm (solid alloy wire) of resistivity 5×10^{-5} ohm-cm. Thus the resistance present in the loop when the wire is in the normal state is 2×10^{-2} ohm, while the inductance of the loop is $2l\ln w/r\times10^{-9}$ hy or 5×10^{-10} hy. Thus the transient currents die in a time $L/R=2.5\times10^{-8}$ second, which is therefore the time required to store information in the cell. The experiments of the accompanying paper (page 294 in this issue) have been performed not with wires but with films. If we take the bar 10^{-4} cm thick, 10^{-2} cm wide and of the above resistivity, the inductance is essentially unchanged, but the resistance is now ~ 6 ohms and the time constant 10^{-10} second. This settling time is very much shorter than times achieved in core memories and illustrates some of the advantages of superconducting devices. Drive currents of 300 ma have been used, which yield, with a pulse width of 10^{-8} second (for convenience in viewing), a signal output $V_s=\frac{LI}{t}\sim\frac{5\times10^{-10}\times0.3}{10^{-8}}=$ 15 mv into the signal line of characteristic impedance ~5 ohms.

These memory cells have been constructed by multilayer vacuum evaporation on a superconducting shield film, the active parts of the cell being less than one mil thick, but the whole, including the support, being for mechanical reasons a few mils thick. This early superconducting memory thus exhibits an information density in three dimensions (if we use 1 mm bit spacing) of 10^4 bits/cc, or 3×10^8 bits/ft^3. There is no obvious reason why a 1 ft^3 memory of this type, containing 10^7 30-bit words, could not be operated with random access in a cycle time of 10 millimicroseconds. The fabrication and assembly problems, however, are not trivial. In a later paper we shall discuss a persistent-supercurrent random-access memory with nondestructive readout.

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*We treat the *isothermal* case — a device in which the wire is in very good thermal contact with the bath — leaving for a later paper the detailed discussion of the interesting and useful effects produced by thermal time constants.