STABLE WIDEBAND EMITTER FOLLOWERS

By Paul J. Bénéteau

ABSTRACT

Emitter followers are analyzed and a simple criterion is established to ensure non-oscillatory conditions. A compensation scheme is proposed to extend the input impedance frequency response, and it is shown that the stability criterion of the compensated case is similar to that of the uncompensated case. Two circuit examples are given: one uses a low-frequency alloy transistor resulting in about a 100:1 improvement in input impedance frequency response, while the other uses kilomegacycle silicon transistors that result in a very wideband amplifier of 8.2 megohms input resistance in parallel with 0.3 pf which is suitable for an oscilloscope preamplifier.

1. INTRODUCTION

Though emitter followers are very widely used to obtain high input impedance circuits, they are usually quite difficult to understand accurately. When they are used at frequencies in the vicinity of or higher than the beta-cutoff frequency, they tend toward oscillatory conditions when operated in a certain fashion. This problem is not new and has been analyzed many times (for example in reference 1); however, these analyses have tended to become somewhat obscured by the mathematics involved and the resulting criteria have not always been clear.

The purpose of this paper is, first, to carry out a simple analysis of conventional emitter-followers; this analysis is then used to define a stability factor K that ensures safe regions of operation for driving capacitive and non-capacitive loads. Secondly, a compensation scheme is presented to improve the input impedance characteristic, and it is shown that the stability criterion is similar to that used for the uncompensated case. Finally, two examples of wideband emitter-follower circuits are given: one uses an alloy germanium transistor with a beta cutoff of about 17 kc in a compensated circuit with an input impedance frequency improvement of about 100:1, while the other uses kilomegacycle transistors in an uncompensated circuit to obtain an input impedance of about 8 megohms in parallel with 0.3 pf which is suitable for an oscilloscope preamplifier.

2. ANALYSIS OF EMITTER FOLLOWERS

a. Uncompensated Case:

An emitter follower is shown in figure 1(a) and its equivalent input circuit in figure 1(b). R_s' is the source resistance, β_o is the low-frequency common-emitter gain, and C_e is the stray (or load) capacitance. The effects of the collector-base capacitances can usually be ignored in the frequencies at

APP-41
September 1961

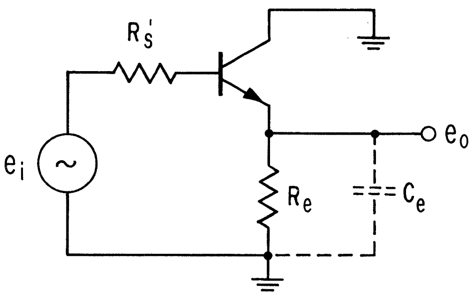

Fig. 1(a) Conventional AC Circuit of an Emitter Follower

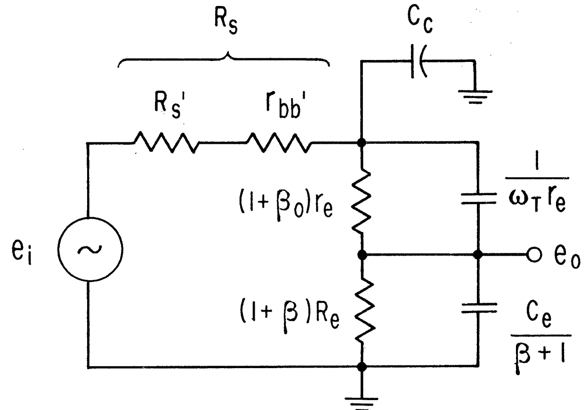

Fig. 1(b) Equivalent Circuit

which emitter followers are used; if it cannot be ignored, it can be simply lumped with $\frac{C_e}{\beta_o + 1}$, since $\frac{1}{\omega_t r_e}$ is a much larger capacitance. This circuit is analyzed in the appendix, and it is shown that

$$\frac{e_o}{e_i} = \frac{\beta_o R_e}{x^2 \frac{R_s}{K} + x\left(R_s(1 + \frac{1}{K}) + \frac{\beta_o r_e}{K}\right) + \left(\beta_o R_e + \beta_o r_e + R_s\right)} \quad (1)$$

where $K = \frac{\omega_e}{\omega_\beta} = \frac{\frac{1}{R_e C_e}}{\omega_\beta}$

and $x = \frac{p}{\omega_\beta} = \frac{j\omega}{\omega_\beta}$,

the only assumption being made that $\beta_o \gg 1$ and that C_c is either negligible or accounted for. This expression can be somewhat simplified by making the assumption that $R_s \gg \beta_o r_e$. With this assumption the expression becomes

$$\frac{e_o}{e_i} \simeq \frac{K \frac{\beta_o R_e}{R_s}}{x^2 + x(1 + K) + \left(1 + \frac{\beta_o R_e}{R_s}\right) K} \quad (2)$$

The poles of this expression are the roots of the quadratic denominator. For no overshoot in the pulse response and a guarantee of no tendency toward an oscillatory condition, it is a sufficient but not necessary condition that these roots be real, i.e., that the discriminant of the denominator be greater than or equal to zero. Letting the discriminant be $\Delta_{unc.}$, the condition is thus that

$$\Delta_{unc.} \equiv \left[(1 + K)\right]^2 - 4\left(1 + \frac{\beta_o R_e}{R_s}\right) K \geq 0 \quad (3)$$

See figure 2.

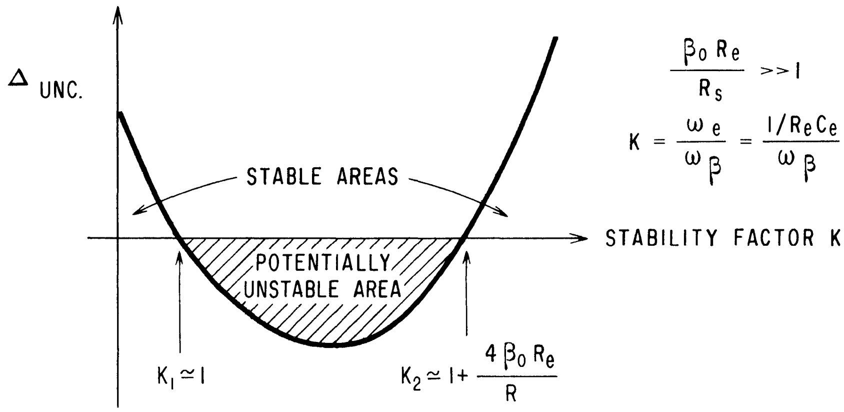

Fig. 2 Graphical Representation of Values of the Stability Factor K Where Nonoscillatory Conditions Are Ensured for the Uncompensated Case

When equation (3) is solved for K, it is found that the roots of Δ are at

$$K_1, K_2 = \left(1 + \frac{2\beta_o R_e}{R_s}\right) \pm \left(\frac{2\beta_o R_e}{R_s}\sqrt{1 + \frac{R_s}{\beta_o R_e}}\right) \tag{4}$$

Generally, if the sourcé is not to be loaded, $R_s \ll \beta_o R_e$; it is to be emphasized that this assumption may not always hold and the exact expression above will have to be used in those cases. The roots are

$$K_1, K_2 \simeq 1, 1 + \frac{4\beta_o R_e}{R_s} \tag{5}$$

It can be seen by reference to Figure 2 that the stable regions of operation are below $K \simeq 1$ or above $K = 1 + \frac{4\beta_o R_e}{R_s}$. For example, if a 600 ohm 100 pf load is to be driven, an f_β of at least $1/2\pi(600)(10^{-10})$ or 2.65 mc/s would be required for safe operation below K = 1. Alternatively, if lower frequency response could be tolerated, and assuming a source impedance of 10K and a β_o of 50, then $K > 1 + \frac{4 \times 50 \times 600}{10,000}$ or 13, which results in an f_β of no more than 130 kc. The frequency and cost requirements would then presumably determine which choice would be taken.

b. Compensated Case

The input impedance of an emitter follower of the type discussed in section 2(a) is approximately βZ_e, where Z_e is the external load impedance. Unfortunately, β is frequency sensitive and falls at 6 db per octave above the beta cutoff frequency f_β. In order to extend the frequency response of the input impedance beyond f_β, it is possible to make Z_e also frequency sensitive but with a rising 6 db slope per octave to compensate for the fall of f_β. See figure 3 where curves of β and Z_e are shown versus frequency for the case $C_e = 0$.

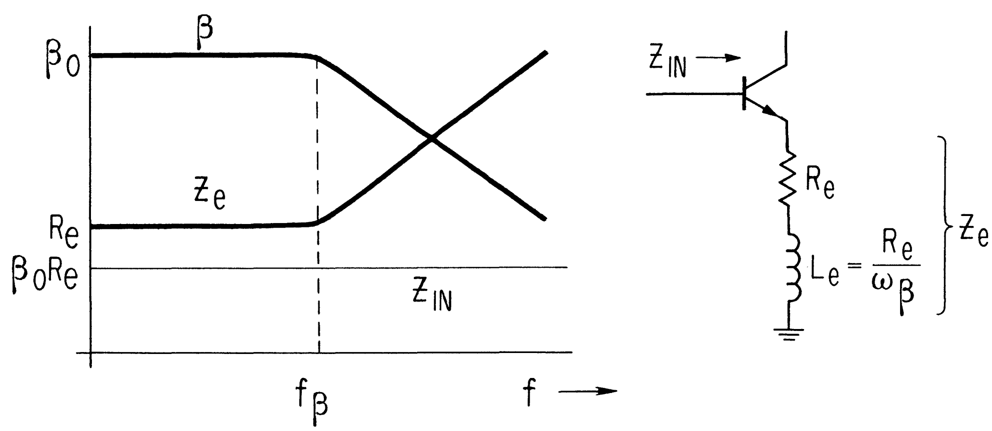

Fig. 3 Emitter Inductance, L_e, Can Compensate for Falling f_β as Frequency Increases

The analysis of the circuit of figure 3 is carried out in the Appendix. It is shown that the reflected load impedance can be expressed as

$$\beta Z_e = \beta_o R_e \left[\frac{p^2 \left(\frac{L_e}{R_e \omega_e}\right) + p \left(\frac{L_e}{R_e}\right) + 1}{p^2 \left(\frac{1}{\omega_e \omega_\beta}\right) + p \left(\frac{1}{\omega_\beta} + \frac{1}{\omega_e}\right) + 1} \right] \tag{6}$$

Using the notation of section 2a, and putting $L_e = R_e/\omega_\beta$,

$$\beta Z_e = \beta_o R_e \frac{\frac{x^2}{K} + x + 1}{\frac{x^2}{K} + x\left(1 + \frac{1}{K}\right) + 1}$$

Unfortunately, all the frequency dependent terms in x do not cancel and therefore, a constant resistive input impedance is not obtained. However, for large K (say >10), the term $1 + {}^1/_K$ approaches 1 and pole-zero cancellation occurs. Moreover, for small K, the expression becomes

$$\beta Z_e \simeq \beta_o R_e \frac{\frac{x^2}{K} + x + 1}{\frac{x^2}{K} + \frac{x}{K} + 1} \qquad (K < 0.1)$$

The argument is now made that if a compensation scheme is used, then the frequency response of the impedance will presumably be carried out considerably past ω_β. Therefore, x will usually be somewhat greater than 1 and the squared terms will tend to predominate. This will leave an impedance of approximately $\beta_o R_e$ which is substantially resistive.

The stability criterion is, therefore, that K be large or small, as previously, with the restriction that, if K is small, x must be large (say 3.2 for 5% assumption accuracy). The required stability factors are summarized in Table I for both the compensated and uncompensated cases.

TABLE I SUMMARY OF STABILITY FACTORS K
($K = \omega_e / \omega_\beta$)

	K must be Less Than	K must be Greater Than
Uncompensated Case ($L_e = 0$)	1	$1 + \frac{4\beta_o R_e}{R_s}$
Compensated Case ($L_e = R_e/\omega_\beta$)	0.1*	10

* The bigger $x = \omega/\omega_\beta$, the better the approximation. For better than 5% approximation, $x > 3.2$.

The voltage gain then becomes simply

$$\frac{e_o}{e_i} \simeq \frac{1}{1 + Z_s/\beta_o R_e} \qquad K \geq 10 \text{ or } K \leq 0.1 \tag{7}$$

as in the low-frequency case. The frequency limiting factor will now presumably be the C_c of the transistor.

3. EXAMPLES OF COMPENSATED EMITTER-FOLLOWERS **

a. Circuit Using a Low-Frequency Transistor.

The circuit of Figure 4a is offered as a demonstration of this compensation method to extend the frequency response of the input impedance of low frequency transistors. The 2N175 used had a measured beta cutoff frequency of about 17 kc. The calculated value of L_e is 1 mh, but as shown in Figure 4b

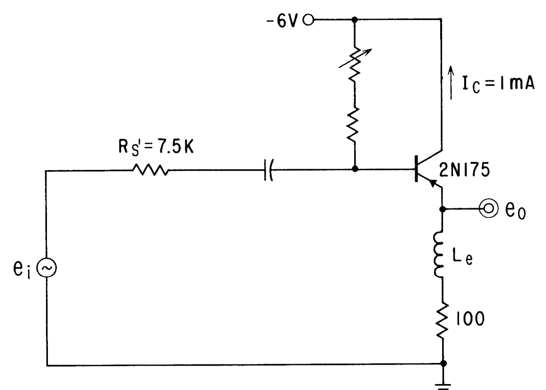

Fig. 4(a) A Circuit Demonstrating the Compensating Method

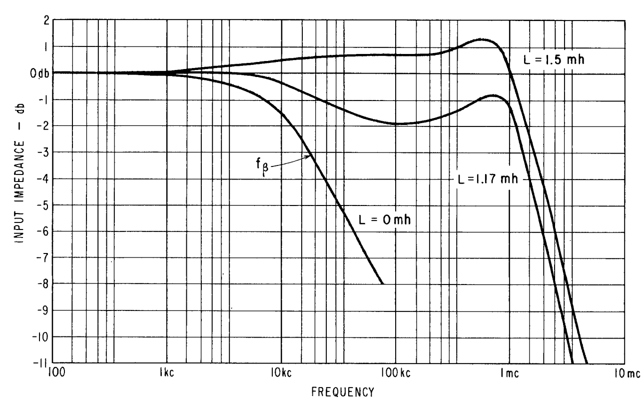

Fig. 4(b) Input Impedance Frequency Response

had to be increased slightly to compensate both for C_c and the self-resonance of the inductor used. The input impedance with L = 0 had a 3 db point of about 17 kc, while the compensated 3 db point was about 1.5 mc, or an improvement of about 100:1 in frequency. The 1.5 mc fall-off is due both to C_c and the self-resonance of the coil. For a 50 pf load, $K \simeq 2000$. The safe value is approximately $K > 8$, so that a very wide safety margin exists.

** The experimental work for this section was done by John A. MacIntosh of Fairchild Semiconductor.

b. Circuit Using Kilomegacycle Transistors:

This circuit (figure 5) is offered as an example of the use of very fast transistors in wideband high input impedance amplifiers suitable, for example, for an oscilloscope input stage. The circuit consists of two cascaded 2N917 emitter-followers, with the emitter of Q_2 returned to the collector of Q_1 to give bootstrapping action. In this manner the effect of the collector-base bias resistors and capacitance is reduced by $(1 - A)$, where A is the gain from input base to output emitter which is less than 1. The collector-base capacitance of Q_1 is already very small (about 1 pf), such that even when the capacitance across the bias resistors is considered, the resulting input capacitance is reduced to about 0.3 pf. The input resistance for the pair of transistors used was 8.2 megohms.

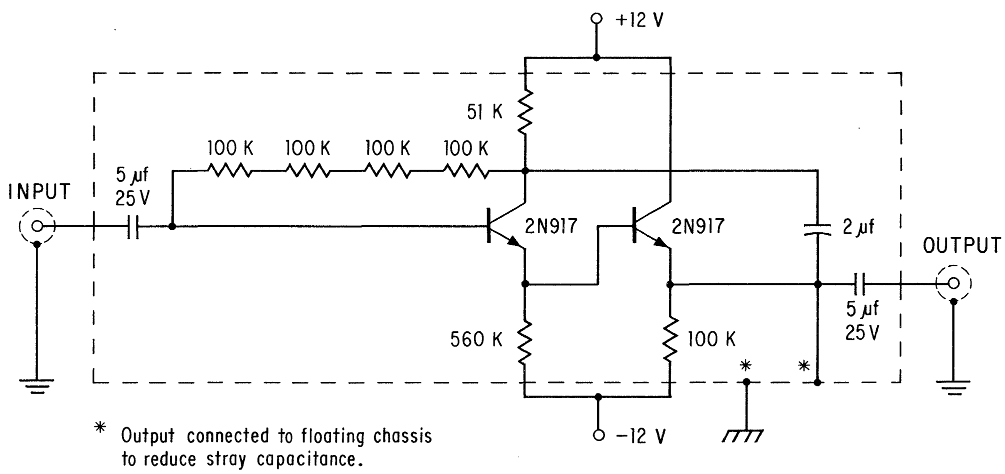

Fig. 5 Wideband High Input Impedance Amplifier.

The voltage frequency response of this amplifier is, of course, extremely wide, and in any practical amplifier will usually be limited by the input RC of the load. The stability factor for the first stage is about 0.003 which is very much less than the limit of 1. The K for the output stage will depend on the load; a 10KΩ 10 pf load would yield a safe value of about 0.1, while a 1KΩ 10 pf load would be marginally stable according to our criterion. For small R small C loads, the value of K would presumably be chosen greater than $4\beta_o R_e / R_s$.

4. CONCLUSIONS

Uncompensated and compensated wideband emitter followers have been analyzed, and a simple stability criterion has been established. Two wideband amplifier designs have been shown. It is felt that these criteria will be a simple and reliable guide for emitter follower designs.

REFERENCES

1) Transistor Circuit Analysis, Joyce and Clark, pp 264-269. Addison Wesley
2) Handbook of Semiconductor Electronics, Hunter, pp 15-36 to 15-38.
3) Two Representations for a Junction Transistor in the Common-Collector Configurations. V.H. Grinich, IRE Transactions on Circuit Theory, Vol. CT-3, Number 1, March, 1956.

APPENDIX

Analysis of an Emitter Follower.

The circuit of figure 6(a) will now be analyzed. The usual case of $L_e = 0$ can then be obtained from the general case.

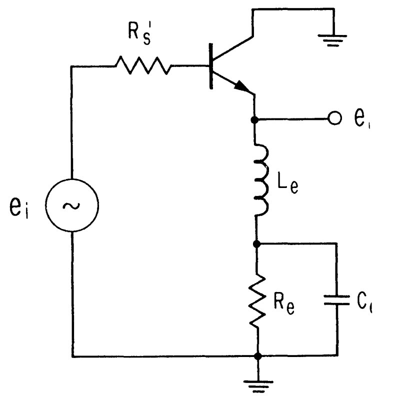

Fig. 6(a) Emitter Follower (AC Circuit)

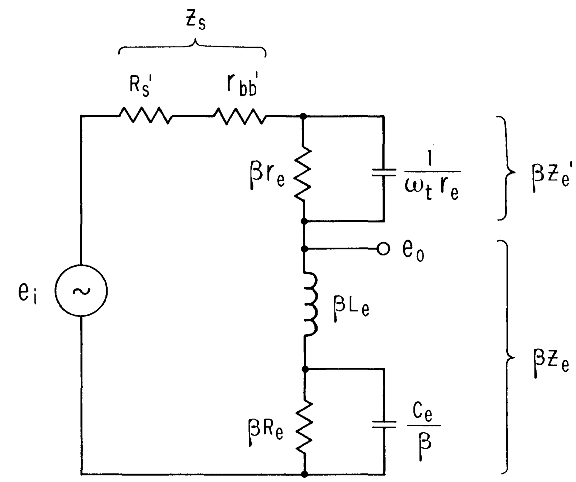

Fig. 6(b) Equivalent Circuit

$$\frac{e_o}{e_i} = \frac{\beta Z_e}{\beta Z_e + \beta Z'_e + Z_s} \qquad \text{A1}$$

$$(\beta Z_e) = \beta L_e\, p + \frac{\beta R_e}{1 + p/\omega_e}$$

or, with $L_e = R_e/\omega_\beta$, $x = p/\omega_\beta$ and $K = \dfrac{1/R_eC_e}{\omega_\beta}$,

$$\beta Z_e = \frac{\beta_o R_e}{1 + x}\, x + \frac{\beta_o R_e}{(1 + \frac{x}{K})(1 + x)}$$

$$= \beta_o R_e \left[\frac{\frac{x^2}{K} + x + 1}{\frac{x^2}{K} + x(1 + \frac{1}{K}) + 1} \right] \qquad \text{A2}$$

$\beta Z'_e = \dfrac{\beta_o r_e}{1 + x}$ and is usually negligible compared to βZ_e.

$$Z_s = R_s' + r_{bb'} = R_s$$

Then, $$\frac{e_o}{e_i} \simeq \frac{\beta_o R_e \left[\frac{x^2}{K} + x + 1\right] \Big/ \frac{x^2}{K} + x\,(1 + \frac{1}{K}) + 1}{\dfrac{\beta_o R_e \left[\frac{x^2}{K} + x + 1\right]}{\frac{x^2}{K} + x\,(1 + \frac{1}{K}) + 1} + R_s}$$

$$= \frac{\beta_o R_e \,(\frac{x^2}{K} + x + 1)}{\beta_o R_e \,(\frac{x^2}{K} + x + 1) + R_s\left[\frac{x^2}{K} + x\,(1 + \frac{1}{K}) + 1\right]}$$

$$= \frac{\beta_o R_e}{R_s}\left[\frac{\frac{x^2}{K} + x + 1}{\frac{x^2}{K}\,(1 + \frac{\beta_o R_e}{R_s}) + x\,(1 + \frac{1}{K} + \frac{\beta_o R_e}{R_s}) + 1 + \frac{\beta_o R_e}{R_s}}\right] \qquad \text{A3}$$

This is the voltage gain expression which reduces to equation 7 if $K < 0.1$ or > 10.

Let us now return to the uncompensated case.

$$Z_e = \frac{\beta_o R_e}{(1 + x)\,(1 + \frac{x}{K})}$$

$$\beta Z'_e = \frac{\beta_o R_e}{1 + x}$$

and $$Z_s = R_s$$

Therefore $$\frac{e_o}{e_i} = \frac{\left(\dfrac{\beta_o R_e}{(1 + x)\,(1 + \frac{x}{K})}\right)}{\left(\dfrac{\beta_o r_e}{(1 + x)\,(1 + \frac{x}{K})}\right) + \dfrac{\beta_o r_e}{1 + x} + R_s}$$

$$\frac{e_o}{e_i} = \frac{\beta_o R_e}{\beta_o R_e + \beta_o R_e\,(1 + \frac{x}{K}) + R_s\,(1 + x)\,(1 + x/K)}$$

which reduces to $$\frac{e_o}{e_i} = \frac{\beta_o R_e}{x^2\,\frac{R_s}{K} + x\left[R_s(1 + \frac{1}{K}) + \frac{\beta_o r_e}{K}\right] + \left[\beta_o R_e + \beta_o r_e + R_s\right]} \qquad \text{A4}$$

which is equation 1 of the paper.