An Algebraic Framework for Uncertain Strips Planning*

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Abstract

This paper presents a formal framework for discrete probabilistic planning so as to extend the classical STRIPS planning framework. Since the semantic of STRIPS relies on the notion of set of formulas describing a situation, uncertainty in STRIPS deals with uncertain sets. An axiomatic theory is described for a new species of set such that membership to these sets can be partial. Then one builds a calculus which handles uncertainty as symbolic probabilistic degrees of membership. The algebra is then plunged in classical planning and usual definitions are given along with an example. The framework holds promise in that it allows non compoundable and non comparable uncertainty (in the uncertain case, two actions may modify the same value in a non comparable manner) and gradual truth degrees.

1 Introduction

Background We refer to classical planning as the STRIPS planning framework. That is to say the STRIPS assumption (i.e. all the facts that are not modified by the performance of an action are assumed to remain unchanged) and its related descriptions (e.g. a state is described with a set of formulas). Note that the "temporal" structure resulting from the planning activity is only an irreflexive order over the action descriptions of the plan: one knows that "1" is before "5" but not at what time. This paper introduces an algebraic framework to handle uncertainty under classical planning. The STRIPS assumption under uncertainty has already been tackled [12] using probability theory where the notion of uncertainty refers to the notion of randomness: a fact is uncertain if we can't decide whether it is true or false (i.e. the probability that the fact is either true or false is equipotent and its value is 0.5 in the unit interval [0,1]).

The main motivation of this paper is to introduce a logical framework to extend the so-called STRIPS planning framework to the uncertain case. This is to say that we want to keep the spirit of STRIPS and just try to answer the following question: how can one handle uncertainty in the STRIPS framework?

Requirements Consider the following example. Let $\mathrm{U}(f)$ denote the uncertainty on the formula f. Let us assume, as in usual probabilistic approaches [6] that the uncertainty on the proposition p, noted $\mathrm{U}(p)$ and the uncertainty on the proposition q, noted $\mathrm{U}(q)$ are respectively α and β. Then if uncertainty is compoundable, $\mathrm{U}(p \wedge q)$ is a function of both $\mathrm{U}(p)$ and $\mathrm{U}(q)$, as in probability: $\mathrm{U}(p\wedge q)=\mathrm{Min}(\mathrm{U}(p),\mathrm{U}(q))$. But if $q = \neg p$ then $\mathrm{F}(\mathrm{U}(p),\mathrm{U}(q)) = \mathrm{F}(\mathrm{U}(p),\mathrm{U}(\neg p)) = \mathrm{F}(\alpha, \beta)$. However, it should be clear that $\mathrm{U}(p\wedge\neg p)=0$: compoundability allows to derive a non null value for $p \wedge \neg p$. Since action application can be seen as an inference process, each performance of an action allows to calculate non null values for the conjunction of p and $\neg p$. Moreover it should be clear that $\mathrm{U}(p)$ may not be comparable to $\mathrm{U}(q)$: the uncertainty of both p and q may be well be quantified although these quantities can be uncertain enough to be comparable. p can be "somewhat" true while q can be "rather" true. How then can we distinguish between those two adverbs? This remark is extremely important when one considers two operators that are partially ordered; if both operators modify the uncertainty of one formula, how can their (uncertain) effects be compared? This entails finally that the description of uncertainty can be gradual. It can range in a discrete manner from "very little" to "very high". One must then provide discrete values and a partial order between these values manipulated within an algebra so that semantic

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can be correctly accounted.

These requirements define a framework to handle uncertainty. But how can it cope with the classical planning framework? In STRIPS the environment is described as a set of formulas. Uncertainty is not a characteristic of the environment but of the agent's beliefs. Consequently uncertainty is a characteristic of all the formulas that can be used to described a state. The performance of an action on such a description must follow the STRIPS assumption and then change only the uncertainty of some formulas while others remain unchanged. A many-valued predicate calculus, built upon the algebra relying on the axiomatic, shall handle these changes.

Outline This paper is organized as follows: the next section presents the logical framework so that it can cope with the (certain) classical planning framework. Section 3 describes how to apply this logic to classical planning along with an example. The last section discusses the framework.

2 An algebraic framework

This section described an axiomatic set theory for a new species of set that can handle uncertain degree. Then, The reader of this section must be familiar with usual axiomatic set theory [4] and classical logic [5]. In order to keep things simple, the algebra presented here can only cope with totally ordered uncertainty degrees (see axiom $\mathcal{A}_3$); the presentation of a partially ordered degree framework is beyond the scope of this paper.

Classical planning describes the environment in a discrete manner where a set of formulas is a discrete situation of the environment. Formulas that belongs to a state describe true facts of the situation. Thus formulas are true in a situation if and only if they belong to the set describing the situation and false otherwise.

2.1 Redefining the notion of set describing a state: uncertain sets

To start with, consider statements of the form "f is S" where f is a formula and S is a state (i.e. a constant denoting a state). Since S is described by a set S, the linguistic statement "f is S" first translates into "$\mathsf{S}(f)$" where S is a predicate symbol[1] and then into the set-theoretical proposition "$f \in S$". However, in the uncertain case, the extension of S is likely to be impossible: describing all the formulas f that could satisfy "$f \in S$" is not tenable because uncertainty potentially makes *all* the formulas belong to S. Consequently, one must extend the usual set theory so that sets can be manipulated as objects (i.e. constants). The concept of *multiset*[2] described below, allows one to exploit and formalize the idea of having sets (and classes in a more general way) to which formulas (and objects in a more general way) belong with a degree. Multisets are the undefined objects of an axiomatic theory which aims at generalizing axiomatic set theory. Providing an axiomatic theory for a new species of set (i.e. multiset) allows to define an algebra that can handle truth values of usual logic connectives (but now in the uncertain case). Note that those truth values are to be modified through inference, *viz.* through the application of action descriptions in some situation (now described as a multiset). If one does not provide such an algebra one should redefine truth values each time different scales of degrees are given: for a planning problem, one could give a scale of membership degree and it might be convenient to give another one for another problem; one must then define the mechanics over the degrees (but independently of the practical values) that can allow to build plans whatever the values (how many values? value of the maximum or minimum, etc) are. Moreover, consider the application of a classical STRIPS action description; this application is possible if and only if the preconditions set is included in the set describing the current situation; and the resulting situation is calculated through set difference and set union. These notions, inclusion, difference and union are lying on the classical axiomatic set theory, the so-called Zermelo-Fraenkel Theory (see [4]), henceforth the ZF-Theory. Defining such a axiomatic for a new kind of set consequently is the first to do.

One first give the language and then the axioms that parallel the usual axiomatic set theory. The axiomatic multiset theory is henceforth noted AMT.

[AMT-Alphabet]

- variables: $x_1, \ldots, x_n, \ldots$
- connectives: $\wedge, \vee, \neg, \rightarrow, \exists, \forall$.
- auxiliary symbols: "(", ")", ",".
- membership symbol: $\in$ which is a ternary predicate symbol with $\in(x_1, x_2, \alpha)$ to be interpreted as x_1 is an element of x_2 with a degree α.
- degree ordering symbol: $\sqsubseteq$ to be interpreted as a total order on the degrees.

[1] It is clear that one should enrich the classical FOL syntax so that well-formed formulas are a parameter of S then so are $\mathsf{S}(f)$. This is exactly what will be done later in the paper when introducing the predicate symbol $\wp$.

[2] The name *multiset* relies on the fact that later on in the paper, formulas shall be attributed a number (i.e. its membership degree) as well as in the usual notion of multiset where numbers denotes multiple occurrences of formulas.

[AMT-Well formed formulas]

- wffs are formed with the previous connectives as in FOL [4].
- if x_1, x_2 and x_3 are variables then $\in (x_1, x_2, x_3)$ is a wff.

The essential features of the previous language are the following. All the individuals are multisets[3], so that, in particular, the degrees are multisets themselves: a multiset is a degree if and only if it is the third parameter of predicate $\in$[4]:

Definition 2.1 ***(The third parameter of ternary $\in$ is a degree)*** $\Delta(\alpha) \leftrightarrow \exists x \exists y (\in (x, y, \alpha))$

Furthermore, the degrees are totally ordered and have a least element denoted $\perp$ and a top element denoted $\top$. Consequently, one manipulates an ordinary set if $\in$'s only third values are either $\top$ or $\perp$:

Definition 2.2 (Ordinary set) *An ordinary set is a multiset μ such that*

$$\forall x \forall \alpha (\in (x, \mu, \alpha) \rightarrow \alpha = \perp \vee \alpha = \top)$$

$\omega(\mu)$ denotes that the multiset μ is an ordinary set.

It should now be clear that multiset-theoretic axioms only differs from usual ZF axioms so as to manage the third parameter of $\in$. AMT contains twelve axioms labelled $\mathcal{A}_1$ to $\mathcal{A}_{12}$; however, for brevity reasons, only the axioms immediately related to planning are presented: **Extensionality** defines equality between multisets; **Pairing** and **Sum-multiset** define multiset union; and **Power-multiset** defines multiset inclusion. Multiset difference needs axioms to define the complement of a multiset and will not be given.

Extensionality ($\mathcal{A}_1, \mathcal{A}_2$) Multisets having the same elements with the same degrees are equal.

Totally ordered membership degree ($\mathcal{A}_3$) If $\Delta(\alpha)$ then α takes its values in a totally ordered set. The structure $\langle \alpha, \leq \rangle$ with $\Delta(\alpha)$ forms a lattice so the join and meet operations respectively are min and max (their definition are obvious and are here omitted). However, the axioms must handle the limit case where $\perp$ and $\top$ together are the degrees as it is when considering ordinary sets (i.e. $\perp = 0$ and the formula does not belong to the set; $\top = 1$ and the formula does belong to the set).

Pairing ($\mathcal{A}_4$) Given any multisets x and y there exists a multiset set z whose elements *exactly* are x *and* y (z is then an ordinary set).

[3] In classical set theory, all the individual are sets.

[4] In classical set theory, a set is said to be an element of another set if and only if the first parameter of the classical membership binary predicate $\in$.

Sum-multiset ($\mathcal{A}_8$) For every multiset x there exists a multiset u, whose elements are those objects occurring in at least one element of x.

Power-multiset ($\mathcal{A}_{10}$) For every multiset y there exists a multiset z, the elements of which are *exactly* the sub-multisets of y (z is then an ordinary set). One immediately has:

Definition 2.3 (Multiset inclusion) *A multiset x is a sub-multiset of the multiset y if and only if $\forall u \forall \alpha (\in (u, y, \alpha) \rightarrow \exists \beta (\in (u, x, \beta) \wedge \beta \leq \alpha))$ and is denoted $x \subset_{AMT} y$.*

Theorem 2.1 (Multiset union) *$y \cup_{AMT} z$ denotes the union of the multiset y with the multiset z if the two following properties hold:*

1. $\in (x, y, \alpha) \wedge \in (x, z, \beta) \rightarrow \in (x, y \cup_{AMT} z, \max(\alpha, \beta))$.
2. $\in (x, y \cup_{AMT} z, \gamma) \rightarrow \exists \alpha (\in (x, y, \alpha) \wedge \alpha \leq \gamma) \vee \exists \beta (\in (x, z, \beta) \wedge \beta \leq \gamma)$.

Finally, one must note that the AMT-axioms are as consistent as the usual set theory axioms:

Theorem 2.2 (Consistency of AMT-axioms) *If the ZF-axioms are consistent then so too are AMT-axioms.*

AMT provides a relational algebra based on the ternary predicate symbol $\in$. However, an *algebra* refers to a formal systems with free variables only (i.e. the variables denoting the individuals of the system); whereas the STRIPS environment description language deals with bound variables [7, 9, 11]. We now need a *calculus*. We thus here introduce a simple predicate calculus (with only one predicate symbol) built upon the algebra that AMT defines.

3 Algebraic uncertainty and classical planning

We have previously extended the notion of membership. We are to introduce symbolic probabilities so that we can introduce uncertainty within the STRIPS framework.

The first section describes the (symbolic) probabilistic extension to multiset and the second one applies the theorems to STRIPS.

3.1 A probabilistic calculus

The following results are obtained by defining a finite cover on $\Delta(\alpha)$ which induces a many-valued calculus which is not of our concern here. The reader is referred to [2, 3] for a full treatment.

Briefly, we add the binary predicate symbol $\wp$ to the previous alphabet which is to be interpreted as follows. If f is a well formed formula of the AMT-Language and $\Delta(\alpha)$ then $\wp(f, \alpha)$ is a well formed formula of the

$\mathcal{A}_1$	:	$\forall y \forall z[\forall x \forall \alpha(\in(x, y, \alpha) \leftrightarrow \in(x, z, \alpha)) \rightarrow y = z]$	Extensionnality
$\mathcal{A}_2$	:	$\forall \alpha \forall \beta[\forall x \forall y(\in(x, y, \alpha) \leftrightarrow \in(x, y, \beta)) \rightarrow \alpha = \beta]$	Extensionnality
$\mathcal{A}_3$	:	$\forall \alpha \forall \beta \forall \gamma[(\alpha \leq \beta \rightarrow \Delta(\alpha) \wedge \Delta(\beta))$ $\wedge(\Delta(\alpha) \wedge \Delta(\beta) \wedge \alpha \leq \beta \wedge \beta \leq \alpha \rightarrow \alpha = \beta)$ $\wedge(\Delta(\alpha) \wedge \Delta(\beta) \wedge \Delta(\gamma) \wedge \alpha \leq \beta \wedge \beta \leq \gamma \rightarrow \alpha \leq \gamma)$ $\wedge(\Delta(\alpha) \wedge \Delta(\beta) \rightarrow \alpha \leq \beta \vee \beta \leq \alpha)$ $\wedge \exists \top(\Delta(\top) \wedge \forall \alpha(\Delta(\alpha) \rightarrow \alpha \leq \top))$ $\wedge \exists \bot(\Delta(\bot) \wedge \forall \alpha(\Delta(\alpha) \rightarrow \bot \leq \alpha))$	Totally ordered membership degree
$\mathcal{A}_4$	:	$\forall x \forall y \exists z(\omega(z) \wedge(\in(p, z, \top) \leftrightarrow p = x \vee p = y)$	Pairing
$\mathcal{A}_8$	:	$\forall x[\exists u(\forall y \forall \gamma(\in(y, u, \gamma) \leftrightarrow \exists z \exists \alpha \exists \beta(\in(y, u, \alpha) \wedge \in(u, x, \beta)) \wedge(\gamma = \max(\alpha, \beta))))]$	Sum-Multiset
$\mathcal{A}_{10}$	:	$\forall y \exists z(\omega(z) \wedge \forall x(\forall p \forall \alpha(\in(p, y, \alpha) \rightarrow \exists \beta(\in(p, x, \beta) \wedge \beta \leq \alpha)) \leftrightarrow \in(x, z, \top)))$	Power-Multiset

Table 1: AMT-Axioms immediately relevant for Classical Planning.

AMT-Language to be interpreted as f has degree of uncertainty α.

Enriching the syntax Then, for any x in the domain of the model $\mathcal{M}$ one imposes on the new predicate $\wp$ the four following axioms (if α is a degree then $\neg\alpha = \top - \alpha$):

1. if $\mathcal{M} \models \phi(x)$ then $\mathcal{M} \models \wp(\phi(x), \top)$.
2. if $\mathcal{M} \models \neg\phi(x)$ then $\mathcal{M} \models \wp(\phi(x), \bot)$.
3. if $\mathcal{M} \models \wp(\phi(x), \alpha)$ then $\mathcal{M} \models \wp(\neg\phi(x), \neg\alpha)$.
4. if $\left|\begin{array}{l} \mathcal{M} \models \wp(\phi(x), \alpha) \\ \mathcal{M} \models \wp(\psi(x), \beta) \\ \mathcal{M} \models \wp(\phi(x) \wedge \psi(x), \bot) \\ \mathcal{M} \models \wp(\phi(x) \vee \psi(x), \gamma) \end{array}\right.$ then $\mathcal{M} \models \wp(\phi(x), \gamma)$ with $\gamma \geq \neg(\beta \rightarrow \alpha)$.

This syntactic sugar leads to the following theorems.

Finite set of values We now make the hypothesis that we have a finite set of n degrees noted 0 to $n-1$.

A classical planning situation is a set of formulas. It is then also a conjunction of formulas. Thus, one needs calculate the global uncertainty value of a conjunction of formulas:

Theorem 3.1 (Conjunction with uncertainty) *If* $\mathcal{M} \models \wp(\phi(x), i)$ *and* $\mathcal{M} \models \wp(\psi(x), j)$ *then* $\mathcal{M} \models \wp((\phi \wedge \psi)(x), k)$ *with* $k \geq \max(i + j - (n-1), 0)$.

One must note that the conjunction of formulas is not compoundable.

The application of an action description is an inference on a situation. One must have an apparatus to calculate such an inference:

Theorem 3.2 (Modus ponens with uncertainty) *If* $\mathcal{M} \models \wp(\phi(x), i)$ *and* $\mathcal{M} \models \wp((\phi \rightarrow \psi)(x), j)$ *then* $\mathcal{M} \models \wp(\psi(x), k)$ *with* $k \geq \max(i + j - (n-1), 0)$.

Some remarks. First, in the planning sense, $\phi(x)$ is of course the conjunction of the formulas of the current situation where α is calculated with the apparatus of theorem 3.1. Second, an action description may have more than one postcondition $\psi(x)$; consequently, the above modus ponens with uncertainty shall be applied to one postcondition at a time. If $\psi(x)$ unifies with a formula, say $f(x)$ which belongs (say, with some degree δ) to the situation where the action description is applied, i.e. $\wp(f(x), \delta)$, one must then consider the disjunction between $\psi(x)$ and $f(x)$: indeed, either $f(x)$ is true with some degree δ in the new situation or it is true with some degree γ in the new situation. One must then have an apparatus to calculate the uncertainty degree of a disjunction between formulas:

Theorem 3.3 (Disjunction with uncertainty) *If* $\mathcal{M} \models \wp(\phi(x), i)$ *and* $\mathcal{M} \models \wp(\psi(x), j)$ *then* $\mathcal{M} \models \wp((\phi \vee \psi)(x), j)$ *with* $k \geq i + j - \max(i + j - (n-1), 0)$.

The need for a disjunction is better understood with the notion of establisher [1]. Suppose two operators can both establish a precondition. Then, there are two (mutually exclusive) ways to achieve this precondition (recall that in the certain STRIPS framework, only one establisher establishes a precondition). Consequently, since in the uncertain case one cannot fully choose between those two, one must admit there is a choice: the disjunction translate this choice.

3.2 Uncertain Strips planning

This section defines a planning framework which is based on the definitions and theorems of the previous sections. This planning framework (only) extends the classical planning framework in the uncertain case with respect to the requirements of the introduction. SPP henceforth denotes this framework.

As an illustration, we will encode the planning problem of finding a completion of a plan in the case of the satisfaction of a boolean formula. This encoding is taken from [10]. Figure 1 gives a graphic representation of this problem. A planning operator is a represented as a square; the preconditions and postcon-

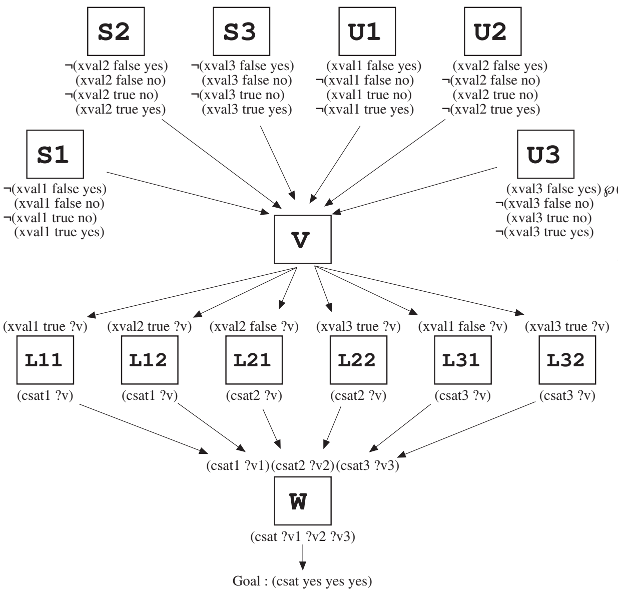

Figure 1: An incorrect partially ordered STRIPS plan to achieve $(x_1 \vee x_2) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_3)$.

ditions are respectively above and under the square. An Arrow denotes a precedence constraint between two operators.

A 2-Sat problem has been chosen in order to keep things simple: 3 operators set the x_i to true and 3 others unset the x_i to false. The L_{ij} operator achieves the jth variable of the ith clause. V has ordering purposes and W holds the 2-Sat formula to be achieved. This example has been chosen because it is easily encoded as a planning problem, it has its logical counterpart, it provides two establishers for the same precondition and the concept of "finding a correct linearization" matches "the probability that a goal is to be satisfied". For this problem, degrees take their values in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Henceforth we refer to this problem as 2-Sat(3,3).

Figure 2 illustrates the encoding that can be derived with the SPP framework presented in this paper; it is used along with the definitions that follows.

Definition 3.1 (Spp Alphabet) *The following alphabet of the* SPP *framework consists of the following symbols:*

- *predicate symbols:* $P_1,\ldots,P_n$ *and the ternary predicate symbol* $\wp$.
- *constant symbols:* $c_1,\ldots,c_n$.
- *variables:* v_0, v_1, v_2,...
- *degrees:* i, j, k,...
- *connective:* $\neg$.
- *auxiliary symbols:* (,), ,.

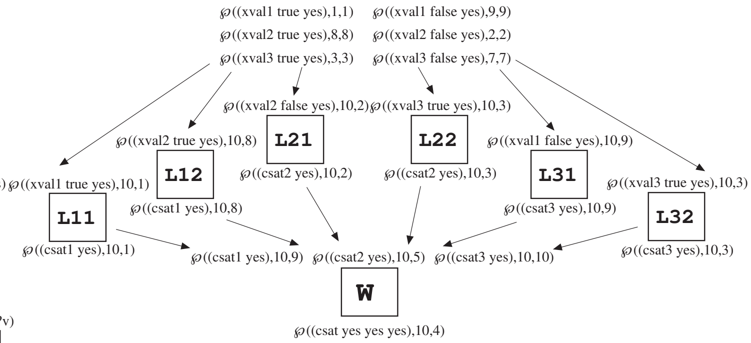

Figure 2: A SPP encoding to achieve $(x_1 \vee x_2) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_3)$.

Recall that classical planning terms are only made of constants or variables; and that classical planning formulas are only made of predicates or the negation of a predicate.

$\wp$ now has three parameters which must be interpreted as follows: f is classical a formula as in the usual STRIPS framework; i_e is the estimated degree of f in a situation and i_c is the degree that has been calculated up to now. Indeed, since we attempt to cope with uncertainty, one must keep track of a desired degree (i.e. for instance, a formula may only need to be "rather" certain for the problem to be solved) and the current degree that has been calculated from the application of action descriptions (which can be fairly different from the estimated degree).

For 2-Sat(3,3), we have:

- 3 unary predicate symbols: csat1, csat2, csat3; 3 binary predicate symbols: xval1, xval2, xval3; and 2 ternary predicate symbols: csat and $\wp$.
- 3 constant symbols: true, false, yes.
- degrees: i, j, k,...
- auxiliary symbols: (,), ,.

Definition 3.2 (Spp Formulas) SPP-*Forms is the smallest set* X *such that if* f *is a classical planning framework formula and both* i_e *and* i_c *are degrees then* $\wp(f, i_e, i_c) \in X$.

For instance, $\wp$((xval1 true yes),1,1) says that there exists a situation to which the formula (xval1 true yes) belongs with the degree 1.

These two degrees together allows to keep track of how far the (calculated) solution is from the desired solution: this is just a way to record partial goal achievement. i_e is estimated degree to which the formula f belongs to a multiset. Thus, a state description is now a multiset to which formulas belong with some degree:

Definition 3.3 (Spp State description) *A* SPP *state description is a multiset of* SPP *formulas.*

Note that if there is no SPP formula $\wp(f, i_e, i_c)$ for the (classical planning framework) formula f in a state description s then the calculated probability that r belongs to s is assumed to be equipotent: $\wp(f, i_e, i_r)$ where i_r is the degree which translates equipotency (which is 0.5 in probability theory). Abusively, one shall note $s \models \wp(f, i_e, i_c)$ when f should belong to s with degree i_e and in fact belongs to s with degree i_c which respectively translates to the AMT notation $\in(f, s, i_e)$ and $\in(f, s, i_c)$.

Definition 3.4 (Spp Action description) *A* SPP *operator $\mathcal{O}$ is a 3-tuple (N($\mathcal{O}$), Pre($\mathcal{O}$), Pos($\mathcal{O}$)) where:*

1. *N($\mathcal{O}$) is a syntactic expression of the form $\mathcal{O}(x_1, ..., x_n)$ where x_1,...,x_n are the only variables that are used in the classical planning framework formulas of Pre($\mathcal{O}$) and Pos($\mathcal{O}$).*
2. *Pre($\mathcal{O}$) and Pos($\mathcal{O}$) both are multisets of* SPP *formulas and respectively are the preconditions and postconditions of $\mathcal{O}$.*

For instance, the operator W (see figure 2):

$$\mathsf{W} = \begin{cases} N(\mathcal{O}) & = \mathsf{W}() \\ Pre(\mathcal{O}) & = \{\wp((\text{csat1 yes}), 10, 10), \\ & \quad\ \wp((\text{csat2 yes}), 10, 10), \\ & \quad\ \wp((\text{csat3 yes}), 10, 10)\} \\ Pos(\mathcal{O}) & = \{\wp((\text{csat yes yes yes}), 10, 10)\} \end{cases}$$

Of course, an operator is applicable in a situation iff its precondition multiset is included in the multiset describing the situation (see definition 2.3 for multiset inclusion) where it is applied. However, it should be obvious that in the uncertain case, any operator is applicable in any situation, with some degree. Thus, if $\wp(f, i_e, i_c)$ and $i_e \geq i_c$ then the operator is applicable in the strict sense of AMT; and $i_e < i_c$ translates the partial achievement of the preconditions. Consequently, the applicability of an action description just records the calculated degree for all its preconditions:

Definition 3.5 ***(Applicability of a Spp Action Description)*** *Let $\mathcal{O}$ be a* SPP *action description. $\mathcal{O}$ is always applicable to any situation description s with the following properties:*

1. *If Pre($\mathcal{O}$) $\models \wp(f, i_e, i_c)$ and $s \not\models \wp(f, i_e, i_c)$ then $s \models \wp(f, i_e, i_r)$.*
2. *If Pre($\mathcal{O}$) $\models \wp(f, i_e, i_c)$ and $s \models \wp(f, j_e, j_c)$ then $s \models \wp(f, k_e, k_c)$ with $k_e = i_e$ and $k_c = j_c$.*

Since we consider that any operator is applicable to any situation, the degree to which this applicability is possible is crucial:

Definition 3.6 ***(Uncertainty on the applicability of a Spp action description)*** *Let $\mathcal{O}$ be a* SPP *action description applicable to a situation description s. $i_{\mathcal{O},s}$ the degree to which $\mathcal{O}$ is applicable to s is (i_{c,p_i} is the calculated degree of precondition p_i and $\mathcal{O}$ possess π preconditions)):*

$$i_{\mathcal{O},s} \geq \max\Big(\Big(\sum_{\in(p_i, Pre(\mathcal{O}), i_{c,p_i})} i_{c,p_i}\Big) - ((\pi - 1) \times (n-1)), 0\Big)$$

For instance, consider operator W in figure 2; according to values in figure 2, $i_{\mathsf{W},s} \geq \max((9 + 5 + 10) - ((3-1) \times 10), 0) = 4$.

Using the uncertain modus ponens, it is now immediate to define the application of an operator:

Definition 3.7 ***(Application of a Spp Action Description)*** *Let $\mathcal{O}$ be a* SPP *action description which is applied to a situation description s. r the resulting situation description of the application of $\mathcal{O}$ to s has the following properties:*

1. *If Pos($\mathcal{O}$) $\models \wp(f, i_e, i_c)$ and $s \not\models \wp(f, i_e, i_c)$ then $r \models \wp(f, k_e, k_c)$ with $k_e = i_e$ and $k_c = i_{\mathcal{O},s}$.*
2. *If Pos($\mathcal{O}$) $\models \wp(f, i_e, i_c)$ and $s \models \wp(f, j_e, j_c)$ then $r \models \wp(f, k_e, k_c)$ with $k_e = j_e$ and $k_c = j_c + i_{\mathcal{O},s} - \max(j_c + i_{\mathcal{O},s} - (n-1), 0)$.*

For instance, in figure 2, the first property applies to operator W and the unique postcondition $\wp((\text{csat yes yes yes}), 10, 10)$ is transformed into $\wp((\text{csat yes yes yes}), k_e, k_c)$ with $k_e = 10$ and $k_c \geq \max(4 + 10 - 10, 0) = 4$. Thus the formula (csat yes yes yes) belongs with the calculated degree 4 to the final situation whereas it should belong to it with the estimated degree 10.

Definition 3.8 (Spp Planning Problem) *A* SPP *planning problem is a 4-tuple $(\mathcal{I}, \mathcal{G}, \mathcal{O}_{\mathcal{P}}, \Delta)$ where:*

1. *$\mathcal{I}$ and $\mathcal{G}$ are multisets that respectively defines the initial and final situation of the problem.*
2. *$\mathcal{O}_{\mathcal{P}}$ is the set of* SPP *action descriptions for the problem.*
3. *Δ is the totally ordered set in which degrees takes their values for the problem; this set follows from the AMT axiom $\mathcal{A}_3$.*

For instance, the totally ordered set $\Delta = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; the final situation of the problem illustrated in figure 2 is $\mathcal{G} = \{\wp((\text{csat yes yes yes}), 10, 10)\}$; and the initial situation is the multiset made the six formulas setting and unsetting the variables to true and false.

In STRIPS a plan is a set of (possibly partially) ordered operators. The precedence constraints between operators reflects the need for all the preconditions of all operators to be established by some precedent operator in the plan. Whereas in the SPP framework, an unachieved precondition is assigned an equipotent probability to be achieved, which entails the application of any operators in any situation.

Definition 3.9 ***(Solution of a*** **Spp** ***Planning Problem)*** *Let* $\Pi = (\mathcal{I}, \mathcal{G}, \mathcal{O}_{\mathcal{P}}, \Delta)$ *be a* SPP *planning problem; and* r *be a situation resulting from the sequence of application of some operators of* $\mathcal{O}_{\mathcal{P}}$ *to* $\mathcal{I}$. *This sequence of operators is a solution to* Π *iff for all* g *such that* $\mathcal{G} \models \wp(g, i_e, i_c)$ *then* $r \models \wp(g, j_e, j_c)$ *with* $j_c \geq i_e$.

The notion of sequence of application of some operators is easily understood as follows. One can consider a set of operators and randomly apply any operator of this set until some point. Note again that this is possible since any operator is applicable to any situation.

4 Conclusion

We believe that the presented framework extend the STRIPS framework in the uncertain case in such a way that the classical STRIPS semantic is extended to the uncertain case. This has first been achieved in defining an axiomatic multiset theory where the notion of membership has been extended to handle partial set membership. Then, an algebra has been constructed where symbolic probabilities have been introduced in order to handle uncertainty.

This work is close to the work on BURIDAN [8] and probabilistic planning [6]. However, we concentrated our work in defining clear semantics so that unusual but useful requirements could be achieved within an extension of STRIPS. For instance, symbolic (i.e. gradual) probabilities, non coumpoundable uncertainty and non comparable degrees can be handled in our framework. But no algorithms have been given whereas the work on BURIDAN is rich in planning mechanisms.

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