An Object-Oriented Representation of Pitch-Classes, Intervals, Scales and Chords: The basic MusES

(revised and extended version)

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Abstract
The MusES system is intended to provide an explicit representation of musical knowledge involved in tonal music chord sequences analysis. We describe in this paper the first layer of the system, which provides an operational representation of pitch classes and their algebra, as well as standard calculus on scales, intervals and chords. The proposed representation takes enharmonic spelling into account, i.e differentiates between equivalent pitch-classes (e.g. C# and D*b*). This first layer is intended to provide a solid foundation for musical symbolic knowledge-based systems. As such, it provides an ontology to describe the basic units of harmony. This first layer of the MusES system may also be used as a pedagogical example for those wishing to apply object-oriented techniques to musical knowledge representation.

Résumé
Le système MusES a comme objectif de représenter les connaissances musicales nécessaires à l'analyse harmonique de séquences d'accords en musique tonale. Nous décrivons ici la première couche du système qui propose une représentation opérationnelle des notes et de leur algèbre, ainsi que des intervalles, gammes et accords. Cette représentation a comme particularité de prendre en compte les problèmes d'enharmonie, i.e. de différencier les notes équivalentes comme Do# et Ré*b*. Cette première couche est utilisée pour l'étude de mécanismes d'analyse harmonique et peut être considérée comme une ontologie des concepts de base de l'harmonie. Le but de ce document est aussi de proposer un exemple non trivial d'application de Smalltalk-80 à l'usage des musiciens désirant se lancer dans la programmation par objets.

An Object-Oriented Representation of Pitch-Classes, Intervals, Scales and Chords

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1. Introduction

Musical Analysis is an ideal field for testing knowledge representation techniques. It involves complex knowledge which is well documented, and many examples are available. Lots of research have been devoted to complex harmonic problems, such as performing complete harmonic analyses of tonal pieces or extracting deep structures in jazz chord sequences.
We focus here on a remarkably simple problem, which, to our knowledge, has yet never been fully addressed. The problem is simply to provide a "good" representation of the algebra of pitch classes, including the notion of "enharmonic spelling", which is so vital to tonal harmony, and a representation of intervals, scales and chords to serve as a foundation for implementing various types of harmonic analysis mechanisms. This problem may be considered trivial compared with more complex problems such as computing Shenkerian analysis of Debussy's pieces, but it has always been solved in ad hoc ways (usually in Lisp), using idiosyncratic representation techniques. For instance, [Winograd 93] emphasises the importance of taking enharmonic spelling into account, but proposes an ad hoc representation of chords as (Lisp) dotted lists. Similarily, [Steedman 84] proposes a solution for performing harmonic analysis of chords sequences but, considers all the entities (chords, intervals or notes) as Prolog-like constants and is interested only in higher level properties of sequences deduced from the mere ordering of their elements.

Our goal here is not only to write a program that solves the problems mentioned above, but also to explicitly represent the underlying mechanisms of pitch-class calculus. This representation is claimed to be natural, and the mechanisms that implement the operations on pitch classes are considered isomorphic to human operations. Pushing this idea to its limit (which we occasionally find ourselves doing), the system described here may be considered as a *substitute for* a first-year text-book of introduction to the basics of harmony. Indeed, many of the mysteries of music notation are explicitly solved here simply because the basic entities and mechanisms of music notation are given an operational status.

We will first spend some time on defining precisely the algebra of alterations in pitch-classes and interval computations (parts 2, 3, 4). These parts are important because they are the foundation of all the system, but they are not the most thrilling. Parts 5 and 6 deal with computations on scales, chords, and scale-tone chords, and should be

[1] The source code of the system presented here may be obtained on the web at: http://www-laforia.ibp.fr/~fdp/MusES.html.

much more exciting to the reader. Finally we show how the system can be easily extended, e.g. to take into account exotic tonalities.

2. Algebra of pitch classes

We are interested in representing *pitch classes*, i.e. octave-independent notes and their relations. For example, *pitch class* C refers to the set of all possible C's (C1, C2 and so on, hence the name pitch-class). In order to avoid confusion - because the word class is very polysemic - we will use the word *note* to refer to pitch classes. This convention is also needed in our context, since we will be speaking of "classes" (in the sense of object-oriented programming) of such notes, and want to avoid talking about *pitch class classes*.
For example, *note* C will actually refer to *pitch class* C, i.e. the set of all Cs (modulo 12). We will not consider *actual notes*, with actual pitch (such as Midi pitches in [1 .. 127]) in this presentation. The extension of our theory to represent actual notes - which may be thought of as "instances" of pitch classes - will be discussed in conclusion, and is rather straightforward once the theory of pitch classes is correctly set.

Here is a wish-list of what a good representation of notes (read pitch-class) should take into account :

-A note has a unique name. There are conceptually 35 different notes : 7 naturals, 7 flats, 7 sharps, 7 double sharps and 7 double flats. The unicity of notes is actually very important. There is only one occurrence of each note (in our octave-independent context). Practically, this means that, for example, the minor second of B (C) is *physically* the *same* note as the minor seventh of D, and so on.

- There is a non trivial algebra for notes. The notes are linked to each other half-tone or tone wise, and form a circular list. But some notes are *pitch-equivalent*, (e.g. A# and B*b* , or C##, D and E*bb*). Although the ability to differentiate between equivalent notes may not seem important at this point, it becomes a crucial point when doing harmonic computations. There is a subtle difference between C# and D*b* which actually appears only when *scales* come into play : for instance, the major scale built from C# contains no C, whereas the major scale built from D*b* does contain a C. Stated differently, the names of notes contain condensed harmonic information that are required by harmonic analysis techniques. Our theory should be able to interpret this information.

- There is an non trivial algebra of alterations, which includes the following equations :

o *b* = *b* o # = identity.
For any x in (#, *b*, natural), x o natural = natural.

This algebra is non trivial because not everything is allowed, e.g. triple sharps.

- Notes are linked by the notion of *interval*, which, in a way, *preserves* this algebra. For instance, the diminished fifth of C is not the same note as the augmented fourth of C, but the two notes are equivalent.

- Certain intervals are forbidden for certain notes : for example, the diminished seventh of C*b* does not exist (it would be B *bbb* !).

- Certain scales do not exist, by virtue of the preceding remarks : G# major is impossible (because it would contain a F## in its signature). The same holds for D*b* harmonic minor, and so on.

Although it is certainly possible to write a global algorithm in any procedural language (such as Pascal or Lisp) that takes all these cases into account, there is clearly here a better solution. This solution is based in *abstract data types*, and consists in considering all these 35 notes as a collection of *instances* of various *types*, each type having its own structure and set of *operations*. This approach not only yields a simple implementation, but also provides us with a clear understanding of the operations on pitch classes.

3. Notes as abstract data types

The main idea underlying our representation paradigm is to model the world as a collection of *abstract data types*, i.e. we do not separate operations on one hand and data structures on the other, but rather try to define types (or classes in object-oriented programming) which gather structures and operations. The theory of *algebraic data types* gives a formal framework to represent abstract data types and the formal properties of relations. Abstract data types and object-oriented programming are particularily well suited to represent musical knowledge (Cf. for instance [Smaill&Wiggins 90] who use abstract data types to represent "constituents" useful for analysis, or [Pope 91] who use Smalltalk for sound editing and real-time algorithmic composition). As it turned out, the problem of representing notes and their algebra is a prototypical example as it fits nicely in this formalism.

This approach leads to us to considering the notes as follows :

- All (35) notes are not equal. Some operations are permitted on some notes and not on others. There are 5 different types of notes : NaturalNotes, SharpNotes, FlatNotes, DoubleFlatNotes and DoubleSharpNotes. It is interesting to distinguish different types of notes because its gives a precise definition to alterations : the #, *b* , and natural, may then be seen as *polymorphic functional operations* on types.
For example, the # operation maps the `NaturalNotes` to `SharpNotes` : A# is then seen as the result of operation # to note A (instance of `NaturalNote`), which yields an instance of `SharpNote`, i.e. :

sharp : `NaturalNote` -------> `SharpNote`
sharp(x) is written x#.

This operation is polymorphic because there are actually several distinct sharp operations, depending on the type of the argument. An other # operation maps `SharpNotes` to `DoubleSharpNotes` (e.g. A## = sharp (A#)), and an other one maps `FlatNotes` to `NaturalNotes` (A*b* # = sharp (A*b*) = A), and `DoubleFlatNotes` to `FlatNotes`.

Some operations are common to all note types (e.g. the operation natural), other are specific to one type of note (e.g. the operation *followingPitch* that links C to D, D to E and so on, is valid only for natural notes) and other to a group of note types (e.g. the *sharp* operation is valid for all note types except `doubleSharpNotes`).
Similarily, the *natural* operation is simply identity when applied to NaturalNotes (A natural = A), but is quite different when applied to SharpNotes (A # natural = A) and still different when applied to double SharpNotes (A ## natural = A). This polymorphism of the natural operation is naturally captured by abstract data types.

3.1. From Abstract Data Types to Object-Oriented Programming

Although the theory of abstract data types sheds a new light on the algebra of pitch-class, it does not allow us to write a completely *operational* specification of the mechanisms. We could write out all the axioms of the algebra of pitch-classes and intervals but this is not what we will do now. We will consider a variant/descendant of this formalism, namely object-oriented programming and Smalltalk-80. Object-oriented programming is based on this very idea of defining abstract entities that gather structure and operations in the context in programming languages.
The vocabulary here is a little bit different : Types are called *classes*. Classes define structure in terms of *instance variables* (or slots, attributes). Each class also has a set of *methods*, which are the operations understood by its *instances*. Polymorphism in object-oriented languages is naturally present, since several classes may have different methods having the same name. An important feature of object-oriented programming is the *inheritance* mechanism between classes, that allows factoring common structure and behavior.

In this document, methods will be written with the following format :

```
! aClassName methodsFor: aProtocol!
aMethodName and its arguments
the text of the method.
```

Where `aProtocol` is simply a set of related methods for a particular class. The text of the method is a set of expressions. Each expression is a message sent of the form: `object messageSelector arguments` (Cf. [Goldberg&Robson 89] for further details about Smalltalk).

We will describe the main methods of the system, but not all of them ! The reader whishing to try the system out may obtain the Smalltalk source code by e-mail.

3.2. The hierarchy of notes

In order to represent notes according to these requirements, we define a hierarchy of classes as follows. Each class has its set of instance variables and operations :

1. `Note` represents the root of all classes representing note. It is an abstract class and has no instance variables.

2. `NaturalNote` represents natural notes. There are 7 instances of this class, representing the 7 natural notes (A, B, C, D, E, F, G). Natural notes form the core of the system :

   -They have a *name*, (actually they have two names, to allow French terminology : A = La, B = Si, etc...). The name is used for global access and printing.
   - They are linked to each other according to the order (A, B, C, D, E, F, G). This is represented by two instance variables : *following* and *preceding*, that point respectively to the following and preceding natural note,
   - Moreover, in order to represent the various intervals between notes, we assign to each natural note an arbitrary *semiToneCount*, so that, e.g. semiToneCount(A) = 1, semiToneCount(B) = 3, .., semiToneCount(G) = 11. This semiToneCount is used for interval computations (Cf. method `alterate:toReach`).

- Finally, there are two pointers towards the *sharp* and *flat* notes generated by the natural notes. They represent the function sharp (resp. flat), which maps `NaturalNotes` -> `SharpNotes` (resp. `FlatNotes`). These notes are instances of `SharpNote` (resp. `FlatNote`) (Cf. below).

The structure of class `NaturalNote` is therefore :

```
Note subclass: #NaturalNote
    instanceVariableNames: 'name nom semiToneCount following preceding sharp flat'
```

Class `NaturalNote` defines methods to access following, preceding, sharp and flat notes. These 4 methods are simple accessing methods : their result is the value of the corresponding note. These values are assigned once, at initialization time (Cf. initialization of notes). For instance, the method sharp is defined as :

```
!NaturalNote methodsFor: 'accessing'!
sharp
    ^sharp
```

3. `AlteredNote` is the root of the classes representing altered (and doubly altered) notes. It is an abstract class. It defines only one instance variable called *natural* pointing back to the natural note it comes from. For instance, A#, A##, A*b* , and A*bb* all have A as their *natural*.

4. Finally, there are 4 subclasses of `AlteredNote` for representing respectively sharp, flat, doubleSharp and doubleFlat notes. These classes implement the methods sharp, flat and double flat so as to respect the natural algebra of alterations. For instance, class `FlatNote` implements the following `sharp` method :

```
!FlatNote methodsFor: 'accessing'!
sharp
"my sharp is simply my natural note"
    ^natural
```

Conversely, for sharp notes, the flat operation is defined as the natural operation :

```
!SharpNote methodsFor: 'accessing'!
flat
"my flat is simply my natural note"
    ^natural
```

For `DoubleFlat`, the `sharp` method will consist in delegating the message to the natural note :

```
!DoubleFlatNote methodsFor: 'accessing'!
sharp
"x bb # = x b"
    ^natural flat
```

Method `flat` in `DoubleSharpNote` is similar.

Finally, we need to represent the functional link between a flat (resp. sharp) note and its corresponding doubleFlat (resp. doubleSharp). This is realized by defining an instance variable in class `FlatNote` pointing to the corresponding doubleFlat note (and idem for sharp).
Thus, the method flat is implemented as a simple access method for `FlatNote` (idem for sharp in class `SharpNote`).

To conclude, here is the list of all the implementations of the flat method (the same mechanism applies for the `sharp` operations) :

```
!NaturalNote methodsFor: 'alterations'!
flat
    ^flat

!FlatNote methodsFor: 'alterations'!
flat
    ^flat

!SharpNote methodsFor: 'alterations'!
flat
    ^natural

!DoubleSharpNote methodsFor: 'alterations'!
flat
    ^natural sharp
```

Note that the flat operation is intentionaly not defined for class `DoubleFlatNote`. The flat message sent to a `DoubleFlatNote` will raise an error, which is conform to our philosophy. Idem for method sharp in class `DoubleSharpNote`.

3.3. Equivalence of pitches

Last, we introduce a method for testing the equivalence of pitches. This method, called `pitchEquals:` tests the semiToneCount, and allows to represent the equivalence of certain notes. This method is implemented as follows :

```
!Note methodsFor: 'testing'!
pitchEquals: aNote
        ^self semiToneCount = aNote semiToneCount
```

To implement method semiToneCount, we will once again use polymorphism. The method is defined as follows in the 5 classes :

```
!NaturalNote methodsFor: 'access'!
semiToneCount
"a simple acess method"
    ^semiToneCount

!FlatNote methodsFor: 'access'!
semiToneCount
    ^natural semiToneCount - 1

!SharpNote methodsFor: 'access'!
semiToneCount
    ^natural semiToneCount + 1

!DoubleSharpNote methodsFor: 'access'!
semiToneCount
    ^natural semiToneCount + 2

!DoubleFlatNote methodsFor: 'access'!
semiToneCount
    ^natural semiToneCount - 2
```

Now all the note classes have been defined, and the algebra of pitch is correctly represented. The note classes form the following inheritance tree (instance variables are between parenthesis, and inheritance is represented by indentation) :

```
Object ()
    Note ()
        NaturalNote (name following preceding sharp flat semiToneCount)
        AlteredNote (natural)
            SharpNote (sharp)
            FlatNote (flat)
            DoubleFlatNote ()
            DoubleSharpNote ()
```

Figure 1 represents the class hierarchy as well as the instances A, B, A#, A##, A*b*, and A*bb*, and their relationships.

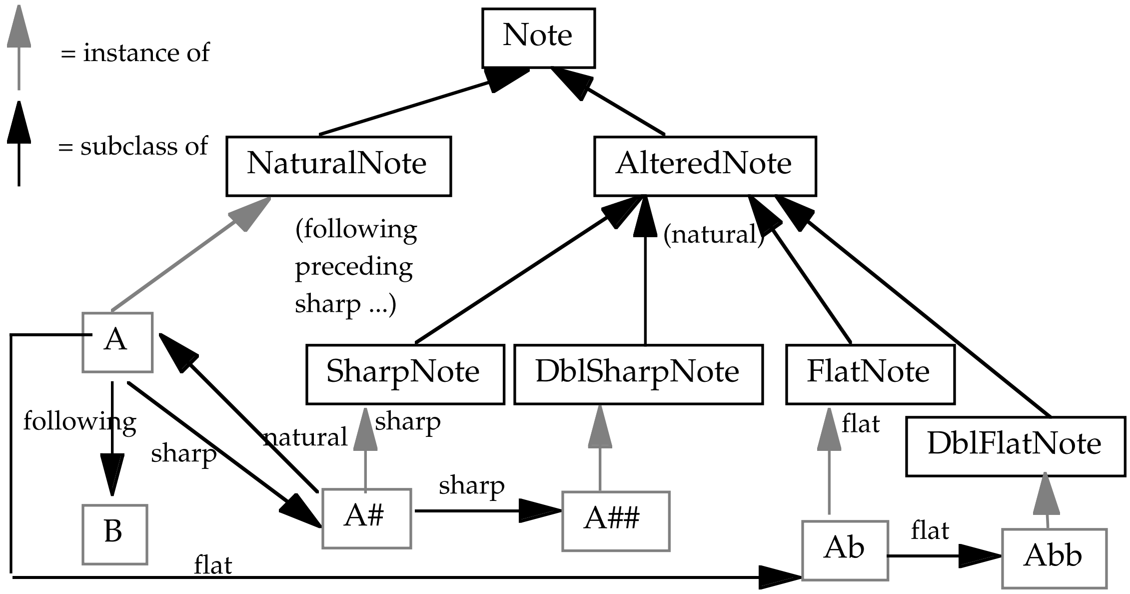

Figure 1. Relationships between several notes.

3.4. Note creation and initialization

Once these classes are defined, we define an initialization method as a class method for `Note`. This method will create the 35 instances of notes and link them according the instance variables defined above. Here is an outline of the method (dot ... are used to avoid repetition for all notes):

```
!Note class methodsFor: 'note initialization'!
initialize
    | as bs .. af bb .. ass bss .. aff bff .. |
    A := (NaturalNote new) semiToneCount: 1; name: #A.
    B := (NaturalNote new) semiToneCount: 3; name: #B...
    as := SharpNote new natural: A.
```

```
bs := SharpNote new natural: B...
af := FlatNote new natural: A.
bf := FlatNote new natural: B...
ass := DoubleSharpNote new natural: A.
bss := DoubleSharpNote new natural: B...
aff := DoubleFlatNote new natural: A.
bff := DoubleFlatNote new natural: B...
A following: B; preceding: G; sharp: as; flat: af...
as sharp: ass. bs sharp: bss...
```

Since notes are unique, we want to have a global access to them. This global access is realized by 7 class variables (`A` to `G`) which point to the corresponding natural notes created during the initialization phase. A set of special methods are written to access these natural notes by messages such as A, B, C (or do, re, mi). Altered notes are then accessed by sending appropriate alteration messages to natural notes.

Here is a micro session that illustrates note access[2].

```
Note C                                      -> C
Note C sharp                                -> C#
Note C sharp sharp flat                     -> C#
Note C flat flat flat    -> error: 'flat' not understood by class DoubleFlatNote
Note C sharp pitchEquals: Note D flat       -> true
```

4. Intervals

Now that notes and the algebra of alterations are correctly defined, interval computation is easy to add (and more interesting !). The same kind of requirements that hold for notes hold for intervals, namely the possibility of differentiating synonymous intervals. For instance, we want to be able to distinguish the *diminished fifth* of C (which is G*b*) from its *augmented fourth* (which is F#, a pitch-equivalent of G*b*).
There are a number of things one can do with intervals, which are :

- computing the top or bottom lacking extremity of an interval, given a note (e.g. *what is the major third of C*, or *what is the note whose major third is C*),
- computing an interval given two notes. For example, we want to be able to answer the question : *what is the interval between C and F# ?* (the answer here is *an augmented fourth*),
- performing some computations on intervals themselves, such as :
  adding intervals (e.g. a major third + a perfect fifth = a major seventh)
  computing *reverse* intervals (the reverse of an augmented fourth is a diminished fifth).

In order to do so, we must have an explicit representation of intervals, that supports those operations. The class `Interval` is defined with the following structure :

---

[2] Note that the instance of `SharpNote` that represents C# is *accessed* by sending the message sharp to the note C, but *prints itself* as C#.

a *type*, which indicates how many notes should be enumerated. The type is represented by an integer (e.g., 2 for a second, 3 for a third, and so forth),
a number of *semiTones*, that represents its actual width (also an integer).

These two informations are sufficient to actually compute the real name of the interval. For instance, a major third interval is represented by an instance of Interval whose *type* is 3 (for 'third'), and whose *semiTones* is 5. This is represented by the printing method of class `Interval`, that prints an interval according to the human (mysterious) terminology, that allows *perfect* fifths ou fourths, but *major* and *minor* thirds:

```
!Interval methodsFor: 'printing'!
printOn: s
    type = 2 ifTrue: [s nextPutAll: (#(diminished minor major augmented) at: (semiTones + 1))].
    type = 3 ifTrue: [s nextPutAll: (#(minor major) at: (semiTones - 2)) ].
    type = 4 ifTrue: [s nextPutAll: (#(diminished perfect augmented) at: (semiTones - 3)) ].
    type = 5 ifTrue: [s nextPutAll: (#(diminished perfect augmented) at: (semiTones - 5)) ].
    type = 6 ifTrue: [s nextPutAll: (#(minor major augmented) at: (semiTones - 7)) ].
    type = 7 ifTrue: [s nextPutAll: (#(diminished minor major) at: (semiTones - 8)) ].
    s nextPutAll: ' ', (#(octave second third fourth fifth sixth seventh) at: type).
```

4.1. Methods to access constant intervals

To ease access to commonly used intervals, we define a set of methods that instantiate class `Interval` accordingly.

Here are some of these methods that speak for themselves :

```
!Interval class methodsFor: 'constants'!
fifth
    ^self type: 5 semiTones: 7
```

```
!Interval class methodsFor: 'constants'!
diminishedSeventh
    ^self type: 7 semiTones: 9
```

4.2. Computing interval extremities

In order to compute the note forming a given interval with a given note, we will follow the human algorithm which says that computing an interval consists in the following steps : (we will take the example of computing the diminished fifth of C*b*) :

1. getting to the natural note. In our example, C*b* yields C.
2. enumerating as many steps as the interval says. Here, a diminished fifth is a fifth, so we will enumerate five notes, starting from C : C, D, E, F, G. We get a G.
3. Adding one or two # or *b* to the resulting note (here G) to yield the right number of half-tones. In our example, we want a diminished fifth, which is 6 half-tones. From C*b* to G there are 8 half tones, so we send the message *flat flat* to the result, eventually getting G*bb*.

Here is the corresponding method, which computes the diminished fifth of a note. It is defined in the root class of notes (`Note`).

```
!Note methodsFor: 'intervals'!
diminishedFifth
    ^Interval diminishedFifth topIfBottomIs: self
```

The main method is `topIfBottomIs:` , which is defined in class `Interval` as follows :

```
!Interval methodsFor: 'computing'!
topIfBottomIs: aNote
    "yields the note making the interval self with aNote"
    ^aNote alterate: (aNote nthFollowing: type - 1) toReach: semiTones
```

This method of class `Interval` uses two methods defined in class `Note` : `nthFollowing:` and `alterate:toReach:`.
Method `nthFollowing:` simply yields the nth following note, in the natural ordering :

```
!Note methodsFor: 'intervals'!
nthFollowing: i
    | result|
    result := self natural.
    i timesRepeat: [result := result following].
    ^result
```

Now the main method is actually the method `alterate:toReach:`, which takes two arguments : a naturalNote *n*, and a number of semiTones *s*. The method sends the right number of sharp or flat messages to the natural note to reach an interval with *s* semiTones.
It is important here to note that this method may be sent to any type of note. The action to perform depends on the type of the note so we actually define 5 such methods.
The first one deals with natural notes. The computation is based on the difference between semiToneCounts of its extremities. Depending on this difference, the messages sharp and flat are sent to the note passed in parameter.

```
!NaturalNote methodsFor: 'intervals'!
alterate: note toReach: s
    | delta |
    delta := (self semiTonesWithNaturalNote: note) - s.
    delta = 0 ifTrue: [^note].
    delta = 1 ifTrue: [^note flat].
    delta = -1 ifTrue: [^note sharp].
    delta = 2 ifTrue: [^note flat flat].
    delta = -2 ifTrue: [^note sharp sharp].
    ^self error: 'illegal interval'
```

The method `semiTonesWithNaturalNote:` is defined simply as a difference of semiToneCounts mod 12 :

```
!NaturalNote methodsFor: 'intervals'!
semiTonesWithNaturalNote: aNote
        ^aNote semiToneCount - semiToneCount \\ 12
```

Now what happens to non natural notes ? The answer is simple. For `SharpNotes` for instance, the computation consists in delegating the result to the corresponding natural note, and then sending a sharp message to the result, as follows :

```
!SharpNote methodsFor: 'intervals'!
alterate: note toReach: s
    ^(natural alterate: note toReach: s) sharp
```

Similarily, the same mechanism holds for Flat, DoubleFlat and DoubleSharp notes.

The dual problem, i.e. finding the "bottom" of an interval, given its top, is now easily defined as follows, by using the "reverse" of an interval :

```
!Interval methodsFor: 'computing'!
bottomIfTopIs: aNote
    "yields the note from which aNote yields interval self"
    ^self reverse topIfBottomIs: aNote!
```

4.3. Computations on intervals

The reverse of an interval is trivially defined by computing the complement to 9 for type, and to 12 for semiTones :

```
!Interval methodsFor: 'reverse'!
reverse
    ^self class type: (9 - type) semiTones: (12 - semiTones)
```

Adding intervals is as easy to do, by simulating an actual computation starting for instance in C. Note that this adding operation is not well defined, because all the combinations do not yield valid intervals. For instance a minor second + a minor second would yield a theoretic *diminished third*, which does not exist. Our + method does not perform any test at this point, and in these illegal cases will yield an interval that cannot print itself ! (this method is just here for fun) :

```
!Interval methodsFor: 'arithmetics'!
+ anInterval
    | note1 note2 |
    note1 := self topIfBottomIs: Note C.
    note2 := anInterval topIfBottomIs: note1.
    ^Note C intervalWith: note2
```

Here is a micro-session that exemplifies interval computations :

```
Note C flatFifth                  ->    Gb
Note C augmentedFourth            ->    F#
Note C majorThird majorThird      ->    G#
Note C flat minorSeventh          ->    Bbb
Note C flat diminishedSeventh     ->    error: illegal interval

Interval diminishedFifth bottomIfTopIs: (Note F sharp)    -> C
Interval diminishedFifth bottomIfTopIs: (Note G flat)     -> Dbb
```

Interval majorThird reverse	-> minor sixth
Interval perfectFifth + Interval majorSecond	-> majorSixth
(Note C diminishedFifth) pitchEquals: (Note F minorSecond)	-> true

4.4. Computing intervals from its extremities

Finally, computing an interval from two notes is simple, and implemented by only one method in class `Note` :

```
!Note methodsFor: 'intervals'!
intervalWith: aNote
    | b b2 type |
    type := 1.
    b := self natural.
    b2 := aNote natural.
    [b2 = b] whileFalse:
                [b := b following.
                type := type + 1].
    ^Interval type: type semiTones: (self numberOfSemiTonesWith: aNote)
```

The method `numberOfSemiTonesWith:` is implemented as follows in class `Note`, by cutting the job in three pieces :

```
!Note methodsFor: 'intervals'!
numberOfSemiTonesWith: aNote
        ^self semiTonesWithNatural +
        (self natural semiTonesWithNaturalNote: aNote natural) -
        aNote semiTonesWithNatural
```

The methods `semiTonesWithNatural` and `semiTonesWithNatural:` are implemented respectively in each subclass to yield the correct result, once again using polymorphism.

This method may be used as follows :

Note C intervalWith: Note F sharp	->	augmented fourth
Note C sharp intervalWith: Note G	->	diminished fifth
(Note C intervalWith: Note G) =		
(Note D sharp intervalWith: Note A sharp)	->	true

5. Scales

Let us now proceed with much more exciting matter : scales and chords. We will consider only 7-note scales here. The theory of modern music implicitly distinguishes between *synthetic* scales form so-called *modes*. Modes are scales that can be derived by transpositing a synthetic scale. For example, the major (synthetic) scale (C D E F G A B) may generate 7 different modes (referred to by greek names such as dorian, myxolidian, aeolian etc), by starting the major scale from all 7 possible notes. We do not know of any publicized effort to describe exhaustively all possible diatonic 7-note synthetic scales. [Slonimsky 47] is a attempt to classify scales and melodic patterns according to various divisions of an octave, upon which ornementation designs are built by interpolation, infrapolations and ultrapolations. Although his thesaurus contains around 1500 scales and patterns, his treatment of 7-note scales is not quite convincing. Only 54 scales are given (under the form of "heptatonic arpeggios"), and not all of them are synthetic modes[3].

Slonimsky mentions an attempt by composer Busoni to find new exotic scales (in *Entwurf einer neuen Aesthetik)*. Busoni would have found 113 7-note scales, but no consideration on exhaustivity is made.

An exhaustive account of 7-note scales should not be too hard however. The number of scales starting from C, and containing all 7 notes (and excluding double sharps and double flats) is easy to compute: each note may be either natural, sharp or flat, which yields a total of $3^6 = 729$ scales. Each of this scale may then be transposed in any of the 12 tones. Some of them are not very interesting because they include enharmonic duplicates (e.g. scales starting by C D# Eb ...). Deciding which ones should be considered synthetic and which ones should be considered as modes is less trivial. We did not address this problem yet.

We will address here the problem of representing the notion of a scale, building up from our previous notions of Note and Interval. Strangely, these are extremely simple to represent, once the foundation is set (and solid!). Here are some of the things we will want to do with scales, in the context of harmonic amalysis :

- Find all the scales that contain n given notes,
- Find the signature of scales (number of sharps and flats),
- Compute the notes of a given scale,
- Represent the fact that certain scales are "forbidden",
- Extract scale-tone chords from a scale.

5.1. Definition and creation of scales

We actually have all we need to represent scales : a scale is an ordered list of intervals, starting on a given root note. The class `Scale` is defined with the following instance variables :

a *root* that points to the root note,
a list of *notes* of the scale[4]. This list of notes may be deduced from the root and type as we will see.

---

[3] It is surprinsing to see Slonimsky often referred to as an exhaustive compiler of musical material. Not only his thesaurus is not exhaustive (as Slonimsky himself jokingly remarks at then end of his introduction), but his method for classifying melodic patterns is laborious and never justified. Even Schoenberg, in a rather unconvinced liner note seems to have been cheated: "I looked through your book and was very interested to find that you have in all probability organized every possible succession of notes. This is an admirable feat of mental gymnastics. But as a composer, I must believe in inspiration rather than in mechanics". It seems that Slonimsky acquired a reputation of music radicalism mainly because his works were hardly ever read (See also his redoubtable 1600-page "Music Since 1900").

4 At this point, we do not consider the problem of finding the scale corresponding to a set of notes, as this is handled by successive layers of the system.

Now there are, as we saw, different types of scale : major scales, harmonic minor scales, melodic minor scales and a vast amount of synthetic scales[5]. The type of the scale could be represented by yet an other instance variable. But there is a better solution that allows us to benefit, once more, from the advantages of polymorphism. This solution consists in representing types of scales by different classes. Each class is defined as a subclass of an abstract class `Scale`, and implements the actual definitions (here, the series of intervals) of the corresponding type of scale. in this scheme, instances of these classes represent actual scales, whose type is determined by their class.

Here is how it works. The main creation method is defined as follows, with one argument : the root of the scale. This creation method is also in charge of computing the list of notes and testing the validity of the scale.

```
!Scale class methodsFor: 'creation'!
root: aNote
    |s|
    s := self new root: aNote; computeNotes.
    s isValid ifFalse: [^self error: 'invalid scale'].
    ^s
```

Now the 2 important methods are `computeNotes` and `isValid`, and are defined as follows :

```
!Scale methodsFor: 'computing notes'!
computeNotes
"intervalList depends on the type of the scale. It is defined in each subclass of Scale"
    notes := self class intervalList collect: [:s | root perform: s]!
```
[6]

The actual interval list is defined in each particular subclass of `Scale`. This is the only method needed to define a subclass of Scale. Since this information does not depend on the actual instance of scale which performs the computation, we represent it by a class method. For instance, here are the definition of `Major`, `HarmonicMinor` and `MelodicMinor` scales by their intervalList definition in the corresponding metaclasses:

```
!MajorScale class methodsFor: 'interval list'!
intervalList
    ^#(yourself second majorThird fourth fifth majorSixth majorSeventh)

!HarmonicMinorScale class methodsFor: 'interval list'!
intervalList
    ^#(yourself second minorThird fourth fifth minorSixth majorSeventh)

!MelodicMinorScale class   methodsFor: 'interval list'!
intervalList
    ^#(yourself second minorThird fourth fifth majorSixth majorSeventh)
```

---

[5] These 3 types of scales are sufficient to describe most of standard be-bop Jazz music.
[6] Note the "smart" use of `perform:` to compute the intervals using the interval computation methods.

The validation test consists in checking the absence of any double altered note :

```
!Scale methodsFor: 'testing'!
isValid
    ^(notes detect: [:n | (n isKindOf: DoubleSharpNote) or: [n isKindOf: DoubleFlatNote]]
        ifNone: [nil]) isNil
```

The creation of scales is defined as follows in class `Note` by sending the a creation message to the corresponding Scale class with self as the root parameter :

```
!Note methodsFor: 'scales'!
majorScale
    ^MajorScale root: self
```

Here is a micro-session for scales :

```
Note A flat majorScale                 -> Ab major
Note A flat majorScale notes           -> (Ab Bb C Db Eb F G)

Note C harmonicMinorScale notes        -> (C D Eb F G Ab B)

Note G sharp majorScale                -> error: 'invalid scale'
```

6. Chords

6.1. Definition and creation of chords

Let us proceed with the core of harmonic analysis : chords. We propose here a representation of chords that is based on the representation of notes, intervals and scales defined above, which allows to make various computations such as :

- finding the name of a chord given a set of notes and a root,
- finding all the possible chord interpretations of a set of notes,
- finding the set of notes corresponding to a chord name,
- finding all the possible harmonic analysis of a chord, in various scale classes.

Chords are represented by a class with two main instance variables : a `root`, which is a note, and a `structure`, which is a list of symbols. Chords may be created by sending a message to the root, with the structure as argument, or by sending a message to class `Chord` with the complete string as argument, such as :

```
Note C sharp chordFromString: 'min'         -> [C# min]
Note D chordFromString: ''                  -> [D]

Chord newFromString: 'A min 7 9'            -> [A min 7 9]
```

6.2. A vocabulary for chord names

Chord name vocabularies abound. None of them is complete, as each one is devoted to a particular style of music. Classical music chord names are very precise concerning the relative organization of notes within the chord (inversions), while Jazz-oriented chord names insist on short-cuts for complex chord superstructures (the famous "+" symbol which means something like "add whatever altered interval pleases you. The more the better"). We introduce a grammar for chords that is able to take into account all possible chords, including the most exotic ones. The syntax rules for the chord name are the following:

1. By default, the root, major third and perfect fifth are included, unless otherwise stated (UOS). For instance [A] is A major = (A C# E).

2. min
means a minor third. E.g. [A min] = (A C E).

3. noRoot
means a chord without root, whatever the rest of the structure may be. For example, [C noRoot] has notes (E G), and [C min noRoot] has notes (Eb G).

4. no3
means a chord without third. For example [C no3] is (C G).

5. no5
means a chord without fifth. E.g [C 7 no5] = (C E Bb).
6. no7
means a chord without seventh. E.g. [C 9 no7] = (C E G D).
7. no9
means a chord without without ninth.
8. no11
means a chord without eleventh.
9. no13
means a chord without thirteenth.

10. diminishedFifth
means a chord with diminishedFifth, and no perfect fifth.
11. augmentedFifth
means a chord with augmentedFifth, and no perfect fifth.

12. minorSeventh
means a chord with minorSeventh.
13. majorSeventh
means a chord with majorSeventh.
14. diminishedSeventh
means a chord with diminishedSeventh.

15. suspendedFourth
means a chord with a fourth and no third.

16. diminishedNinth
means a chord with diminishedNinth and minorSeventh.
17. ninth
means a chord with ninth, and minorSeventh, (UOS).
18. augmentedNinth
means a chord with augmentedNinth and minorSeventh (UOS)

19. diminishedEleventh
means a chord with diminishedEleventh, ninth and seventh (UOS)
20. eleventh
means a chord with eleventh, ninth and seventh (UOS)
21. augmentedEleventh
means a chord with augmentedEleventh ninth and seventh (UOS)

22. diminishedThirteenth
means a chord with diminishedThirteenth, eleventh, ninth and seventh (UOS)
23. thirteenth
means a chord with thirteenth, eleventh, ninth and seventh (UOS)
24. augmentedThirteenth
means a chord with augmentedThirteenth, eleventh ninth and seventh (UOS)

25. diminished
means a chord with root, minorThird, diminishedFifth and diminishedSeventh
26. halfDiminished
means a chord with root, minorThird, diminishedFifth and minorSeventh

This vocabulary may represent any combination of notes in a unique way. It does not, however, take note orders into account. No verification is made on the compatibility of the various structure components. For example [A augmentedFifth no5] has no precise meaning. See next sections for examples.

6.3. Deducing the structure from the list of notes

Deducing the structure of a chord from the list of its notes is a purely procedural process. It is represented by a big (and not very elegant) method, that represents a finite automata, testing each possible case. Here is an outline of the method:

```
!Chord methodsFor: 'creation'!
fromNotes: l
    "assumes the first note is the root"
    ^self fromNotes: l root: l first

fromNotes: aList root: r
    | l |
    root := r.
    l := aList  asOrderedCollection.
    notes := l copy.
    structure := OrderedCollection new.

    (l includes: r) ifTrue: [l remove: r] ifFalse: [structure add: #noRoot].

    (self notes: l contains: #(majorThird minorThird fourth)) ifFalse: [structure add: #no3].
    (((self notes: l contains: #(majorThird minorThird)) not) and: [l includes: r fourth])
                  ifTrue: [structure add: #sus4. l remove: r fourth].
    (l includes: root majorThird) ifTrue: [l remove: root majorThird].
    (l includes: root minorThird) ifTrue: [structure add: #min. l remove: root minorThird].
    …
```

6.4. Deducing the list of notes from the structure

The reverse problem consists in finding the list of notes given a particular structure. The computation is performed automatically in a lazy mode, when the notes access message is sent for the first time to a chord:

```
!Chord methodsFor: 'notes access'!
notes
    notes isNil ifTrue: [self computeAllnotes].
    ^notes
```

For example, the notes of a chord may be computed as :

(Chord newFromString: 'C min dim5 aug9') notes	->	(C Eb Gb Bb D#)

The `computeAllnotes` method is implemented as follows:, by successively invoking specialized methods to compute each part of the chord (computeRoot, computeThird, computeSeventh and so forth).

```
!Chord methodsFor: 'notes computation'!
computeAllNotes
"computes the list of notes from the structure. The job is the opposite of method fromListOfNotes. Assumes root is not nil."

    notes := OrderedCollection new.
    self computeRoot; computeThird; computeFifth; computeSixth; computeSeventh;
        computeNinth; computeEleventh; computeThirteenth; computeDiminished
```

As an example, here are two of the specialized note computation methods:

```
computeThird
    (structure includes: #no3) ifTrue: [^nil].
    (structure includes: #sus4) ifTrue: [^notes add: root fourth].
    (structure includes: #min) ifTrue: [^notes add: root minorThird].
    notes add: root majorThird

computeSeventh
    (structure includes: #no7) ifTrue: [^nil].
    (structure includes: 7) ifTrue: [^notes add: root minorSeventh].
    (structure includes: #maj7) ifTrue: [^notes add: root majorSeventh].
    (structure includes: #dim7) ifTrue: [^notes add: root diminishedSeventh].

    (self structureHasEitherOf: #(9 dim9 aug9 11 aug11 13 dim13))
        ifTrue: [notes add: root minorSeventh]
```

Here are some examples of chord name computations using both mechanisms:

(Chord new fromString: 'Re maj7') notes	OrderedCollection (D F# A C#)
(Chord new fromString: 'Re# maj7') notes	OrderedCollection (D# F## A# C##)
(Chord new fromString: 'C') notes	OrderedCollection (C E G)
(Chord new fromString: 'D min 7 dim5') notes	OrderedCollection (D F Ab C)

(Chord new fromString: 'C aug9') notes	OrderedCollection (C E G Bb D#)
(Chord new fromString: 'C aug9 dim5') notes	OrderedCollection (C E Gb Bb D#)
(Chord new fromString: 'C 13') notes	OrderedCollection (C E G Bb D F A)
(Chord new fromString: 'C 13 aug9') notes	OrderedCollection (C E G Bb D# F A)
(Chord new fromString: 'C 13 aug9 no7') notes	OrderedCollection (C E G Re# F A)
(Chord new fromString: 'C halfDim') notes	OrderedCollection (C Eb Gb Bb)
Chord newFromNoteNames: 'C E G'	[C]
Chord new FromNoteNames: 'C E G#'	[C aug5]
Chord newFromNoteNames: 'C F G'	[C sus4]
Chord newFromNoteNames: 'C E G A'	[C sixth]
Chord newFromNoteNames: 'C E A'	[C no5 sixth]
Chord newFromNoteNames: 'C A'	[C no3 no5 sixth]
Chord newFromNoteNames: 'C E G# B'	[C aug5 maj7]
Chord newFromNoteNames: 'C E Gb Bb'	[C dim5 7]
Chord newFromNoteNames: 'C Eb Gb Bb'	[C halfDim]
Chord newFromNoteNames: 'C Eb Gb Bbb'	[C dim]
Chord newFromNoteNames: 'C Eb'	[C min no5]
Chord newFromNoteNames: 'C E F F#'	[C no5 no9 no7 11 aug11]
Chord newFromNoteNames: 'C Gb G#'	[C no3 dim5 aug5]
"A nice chord from A. Holdsworth"	
Chord newFromNoteNames: 'D# F## A C##)'	[D# dim5 maj7]

6.5. Extracting scale-tone chords

An extremely important and interesting feature of scales is their ability to generate the so-called *scale-tone chords*. In a way, the whole mechanics of harmonic analysis is based on this principle (in the other way round, Cf. below).
Generating chords from a scale is an operation that takes two arguments: a number of polyphony *p*, and an interval *i*. The generation of chords consists simply in building (7) sets of notes. Each set of notes (a chord) is built by taking successively each note of the scale, and iteratively (p times) getting its *i* th following note. The classical case is when `i` = 3, so that chords are built by successive thirds. The method that implements this latter case is `generateChordsPoly:`, which only needs the polyphony parameter.

Here is a micro-session that generates chords :

```
Note C majorScale generateChordsPoly: 7     ->

OrderedCollection ([C maj7 9 11 13] [D min 7 9 11 13] [E min 7 dim9 11 13] [F maj7
9 aug11 13] [G 7 9 11 1] [A min 7 9 11 dim13] [B min dim5 11 dim13])

Note D harmonicMinorScale generateChordPoly: 3  ->
OrderedCollection ([D min] [E min flat5] [F aug5,  G min] [A] [Bb] [C# min flat5])
```

6.6. Computing all possible chord names

An interesting problem to solve is the problem of deducing a chord name from a list of notes, without knowing its root. Actually there are two sub-problems: one in which the root is one of the notes in the list, one - more difficult - in which the root is unknown, and may be absent from the list. These two problems are trivial to solve once the chord vocabulary and the two main creation methods are written.

The first problem is solved by method allChordsFromListOfNotes:, which is written as follows:

```
Chord methodsFor: 'examples'
allChordsFromlistOfNotes: aList
    ^alist collect: [:x | self new fromNotes: alist root: n]
```

The second one is trivially represented by method reallyAllChordsFromlistOfNotes: as follows, where allPlausibleRootNotes yields the list of natural, sharps and flat notes:

```
Chord methodsFor: 'examples'
reallyAllChordsFromlistOfNotes: aList
^Note allPlausibleRootNotes collect: [:x | self new fromNotes: alist root: n]
```

These methods are illustrated by the following session, where we compute all possible chord interpretations of the set of notes (C E G):

```
Chord allChordsFromlistOfNoteNames: 'C E G'
  orderedCollection ( [C] [E min no5 no7 no9 no11 dim13] [G sus4 no5 6]

Chord reallyAllChordsFromlistOfNotesNames: 'C E G'
OrderedCollection ([A noRoot min 7 ] [B noRoot sus4 no5 no7 dim9 dim13 ] [C ] [D
noRoot sus4 no5 7 9 ] [E min no5 no11 no9 no7 dim13 ] [F noRoot no3 maj7 9 ] [G sus4
no5 sixth ] [A# noRoot no3 dim5 ] [C# noRoot min dim5 ] [D# noRoot no3 no5 dim9
] [F# noRoot no3 dim5 7 dim9 ] [G# noRoot no3 no5 no11 no9 no7 dim13 ] [Ab noRoot
aug5 maj7 ] [Bb noRoot no3 no5 no7 9 aug11 sixth ] [Db noRoot no3 no5 maj7 aug9
aug11 ] [Eb noRoot no5 sixth ] [Gb noRoot no3 no5 no9 no7 aug11 ] )
```

6.7. Computing possible analysis

Now that we know how to generate scale-tone chords from a given scale, we are, of course, also interested in the reverse operation, which is the at the heart of harmonic analysis : knowing, for a given chord, what analysis it can "support", i.e. what are the scales from which is may be generated, and, for each of these possible scale, what is the *degree* of the chord.

Let us first represent explicitly the notion of `HarmonicAnalysis`, with a trivial representation by two instance variables :

```
Object subclass: HarmonicAnalysis
    instanceVariableNames: 'scale degree'
```

`HarmonicAnalysis` defines a printing method to print itself between brackets {}, and with roman literals :

```
!HarmonicAnalysis methodsFor: 'printing'!
printOn: aStream
aStream nextPutAll: '{', self romanDegree,' 'of ',scale printString,'}'
```

Now the method that computes all possible analysis for a given chord is naturally defined in class `Chord` by adding all the possible analysis in a given scale (i.e. a subclass of Scale), for all possible scales :

```
!Chord methodsFor: 'computing tonalities'!
possibleTonalites
    "In all possible tonalities = all subclasses of Scale"
    | result|
    result := OrderedCollection new.
    Scale allSubclasses do:
       [:aScaleClass |  result addAll: self possibleTonalitiesInScaleClass: aScaleClass].
    ^result

possibleTonalitesInScaleClass: aScaleClass
   | ana scale chords  possibleTonalities|
   self format.
   possibleTonalities := OrderedCollection new.
   scale := aScaleClass root: Note C.
   chords := scale generateChordsPoly: notes size.
   chords do: [:c  |  (c matchWith: self) ifTrue:
      [ana := Analysis new degree: (scale degreeOfChord: c).
      ana scale: (aScaleClass root:
                         (self root transposeOf: (aScaleClass root intervalWith: scale root))).
      possibleTonalities add: ana]].
   ^possibleTonalities
```

Here is the corresponding micro-session :

```
(Chord new fromString: 'C maj') possibleTonalities              ->
 OrderedCollection (
  {IV of G MelodicMinor}       {V of F MelodicMinor}
  {I of C Major}               {IV of G Major}
  {V of F Major}               {V of F HarmonicMinor}
  {VI of E HarmonicMinor} )

(Chord new fromString: 'D min 7 dim5') possibleTonalities   ->
OrderedCollection (
  {IV of F MelodicMinor}       {VII of Eb MelodicMinor}
  {VII of Eb Major}            {II of C HarmonicMinor} )
```

6.8. Genericity and Reusability

One of the main advantages of our approach, besides the clarification it brings to the overall algebra of alterations, intervals and scales, is the fact that all the mechanisms may be extended very easily, mainly by subclassing.

For instance, our representation of scales makes it straightforward to add new types of scales, using inheritance. Introducing a new type of scale consists simply in creating a new subclass of `Scale`, and defining its interval list. The new class is then ready to use.

For instance, let us define the `HungarianMinor` scale as follows :

```
Scale subclass: HungarianMinor

!HungarianMinor methodsFor: 'interval list'!
intervalList
    "example : (C D Eb F G Ab B)"
    ^#(yourself second minorThird augmentedFourth fifth minorSixth majorSeventh)
```

We can right away use all the preceding methods without any modification. For instance, we can compute the new (exotic) set of possible chords generated by this scale as :

```
(HungarianMinor root: Note C) generateChordPoly: 4          ->
OrderedCollection ([C min maj7] [D dim5 7] [Eb aug5 maj7] [F dim5 dim7] [G maj7]
[Ab maj7] [B min dim7])
```

Of course, we will be also able to use this scale for performing exotic analysis, in the successive layers, at a minimal cost !

Here are for example, the possible analysis of a chord, in this new tonality :

```
(Chord new fromString: 'C maj') possibleTonalities  ->
 OrderedCollection (
{V of F HungarianMinor}       {VI of E HungarianMinor}
{IV of G MelodicMinor}        {V of F MelodicMinor}
{I of C Major}                {IV of G Major}
{V of F Major}                {V of F HarmonicMinor}
{VI of E HarmonicMinor} )

(Chord new fromString: 'D min') possibleTonalitiesIn: HungarianMinor ->
OrderedCollection ( {I of D HungarianMinor }  {VII of Eb HungarianMinor })
```

As John McLaughlin (one of the inventor of Jazz-rock, who, among other things, introduced sophisticated and hard-to-analyse harmonic progressions in Jazz) writes in the foreword of [Mahavishnu 76] : *"... Not all of the following synthetic modes and their derivatives have been used in this book. However I have included them for the benefit of the serious music student, because one can find so much hidden within them, particularily in the extraction of their scale-tone chords"*.

Well, the extraction and study of these exotic scale-tone chords and their interactions is now a child's play :

```
!NeapolitanMinor methodsFor: 'interval list'!
intervalList
    "example : (C Db Eb F G Ab B)"
    ^#(yourself minorSecond minorThird perfectFourth fifth minorSixth majorSeventh)
```

!NeapolitanMajor methodsFor: 'interval list'!
intervalList
"example : (C Db Eb F G A B)"
^#(yourself minorSecond minorThird perfectFourth fifth majorSixth majorSeventh)

!DoubleHarmonic methodsFor: 'interval list'!
intervalList
"example : (C Db E F G Ab B)"
^#(yourself minorSecond majorThird fourth fifth minorSixth majorSeventh)

!MajorLocrian methodsFor: 'interval list'!
intervalList
"example : (C D E F Gb Ab Bb)"
^#(yourself second majorThird fourth diminishedFifth minorSixth minorSeventh)

... and so on : McLaughlin gives 16 synthetic modes, which can be all represented similarily. We can now have the full possible analysis for any chord in any scale, and study them by appropriate queries to MusES.

7. Extending the system

7.1. Representing actual octave-dependent notes

As we said in the beginning, our theory only takes pitch-classes into account, and does not differentiate several notes belonging to the same pitch class (*octave-dependent notes*). The first idea that comes to mind to include these actual octave-dependent notes in our system is to have our present notes (instances of the various subclasses of `Note`) *become classes*, in the sense of OOP, so that one can make instances out of them ! For instance, we would like to say that note *C3* is an instance of pitch-class C. And of course pitch-class C would still be an instance of class `NaturalNote` !
This procedure, which consists in raising all the classes and instances one step higher in the instanciation tree is technically possible[7], but raises an ontological problem : What do we want to consider global vs volatile ?
Intuitively, we would like to say that pitch-classes are global objects, but that octave-dependent notes are not. There are two arguments to support this claim : (1) Pitch classes are not too many (35), compared to actual octave-dependent notes (35 * say, 8 octaves = 280 notes !), and (2) there is no reason to decide a priori what are the limits in the octave multiplication : 8 seems a good approximation, but then we will have the problem of deciding what happens to the upper or lower bounds (would we authorize interval computations on these bounds for instance ?). This lead us to consider a representation for octave-dependent notes as instances, and pitch-classes as classes. Because of space limitation, we will not discuss these technical details here.

Insérer ici la description des OctaveDependentNotes, et les modifications à apporter pour les calculs d'intervalles.

[7] But it is not trivial, since metaclasses are not really first-class objects in Smalltalk. However, small extensions to Smalltalk allow the user to have complete control on metaclasses (Cf. the ClassTalk system by [Cointe&Briot 89]).

7.2. Problems not solved

There are a couple of classical problems involving pitch class computation we did not deal with, such as : computing the scale from a list of notes, or : given an incomplete list of notes (of length < 7), compute the list of plausible scales. We hope that our presentation convinced the reader that these extension are trivial to add to the existing system.

7.3. Representing non trivial reasoning

The system presented here achieves its goal, which is to represent the basic harmonic entities necessary to perform sophisticated reasoning. The representation of this reasoning is the main goal of the higher levels of the MusES system, and is described in subsequent documents. The central idea of these extensions is to use a specialized forward-chaining, first-order inference mechanism (NéOpus) with which all the *reasonings* involving the objects defined here are represented. More on this can be found in [Pachet 91], the expertise is described in [Pachet 87] but represented awkwardly, and a forthcoming report will present a version of the system as an extension to the present architecture.

8. Conclusion

The first layer of the MusES system sets the foundations for the study of various harmonic analysic mechanisms. The basic entites of harmony notes, intervals, scales and chords are defined as by set of classes, having a structure and a behavior. Our approach is validated by the "friendly" feel of the overall system and the almost physical presence of the musical entites, that allow the user to think more naturally, and by the reusability of these entities, and their capacity to support extensions.

9. References

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